Simulating WIMP-Nuclei Collisions in CDMS Detectors

Nate Herbert
Master's Defense
Texas A&M
Department of Physics
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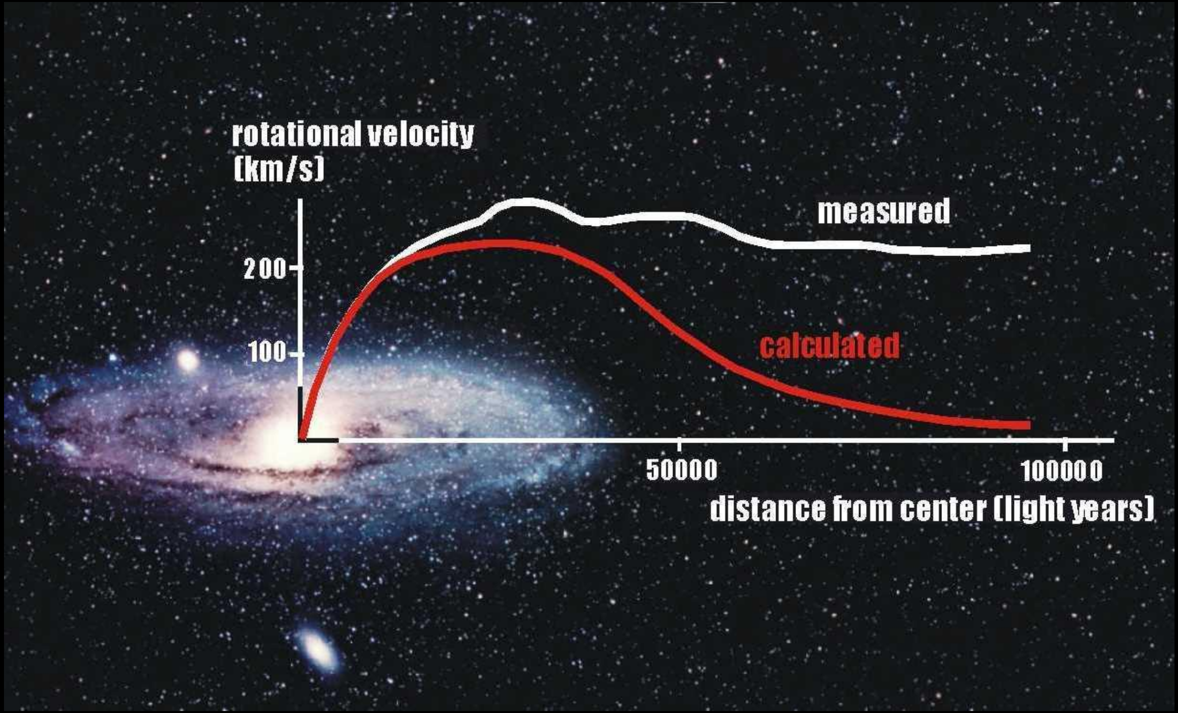
Why Dark Matter?

There are many independent ways to measure mass on a large scale. Results disagree without the inclusion of seemingly invisible mass.

- Rotational velocity of stars in galaxies
- Gravitational Lensing
- X-rays emitted by hot gas in galactic clusters
- Measurements from CMB



First noticed by Fritz Zwicky. 1933

What is Dark Matter?

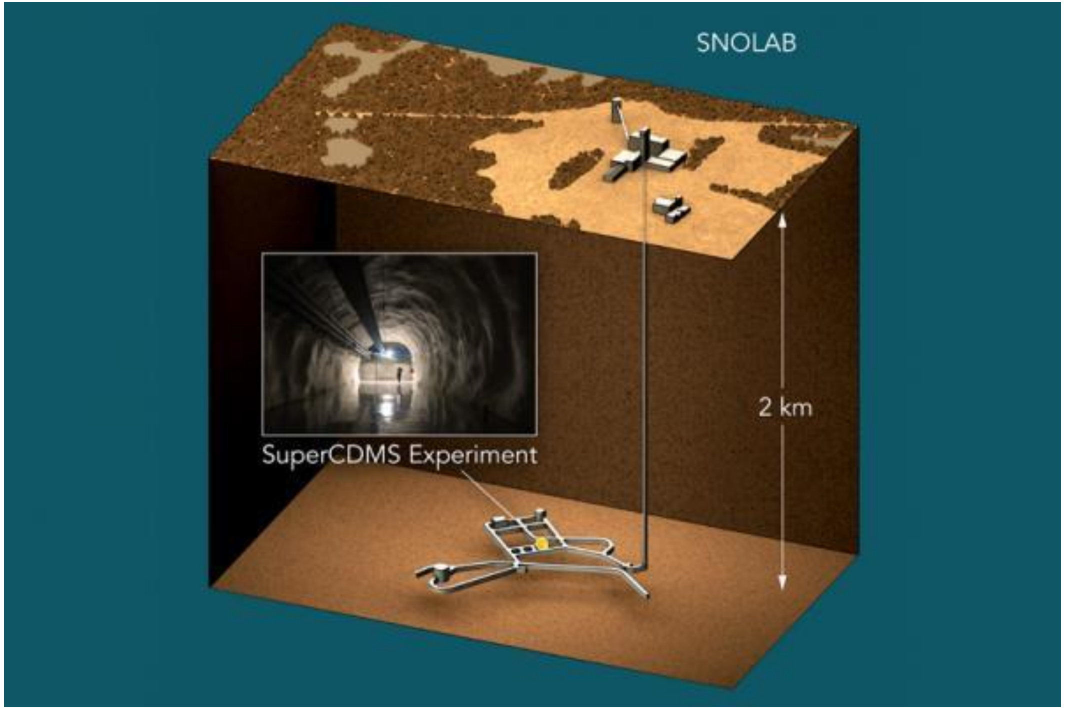
There are many theories to the origin of this extra mass

- MACHOS
- Neutrinos
- Modified Newtonian Dynamics
- WIMPs
- Axions

The bullet cluster shows strong evidence that dark matter is a particle.

Bullet Cluster - NASA 2004

Searching for WIMPs: The Cryogenic Dark Matter Search



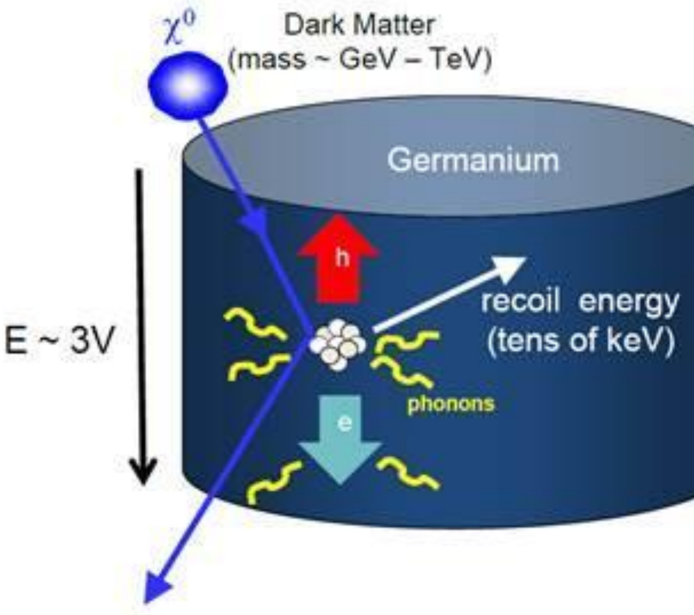
- SuperCDMS SNOLAB is the 4th rendition of an experiment designed to detect WIMP interactions

- The experiment is placed deep underground to prevent as much background as possible

- Shielding is also placed around the detector itself to prevent contamination

- The experiment is sensitive to nuclear and electron recoils that take place within its detectors

How WIMPs Interact with the Detector



- As earth travels through space, WIMPs may occasionally interact with normal matter
- A collision between a WIMP and a nucleus would deposit energy into the detector. Depending on the mass and velocity of the WIMP, and mass of the recoil detector nuclei, we expect a range of recoil energies
- This energy then propagates through the detector where it is eventually collected and recorded

Purpose of This Study

To study the detector response, we simulate particle interactions, energy deposits, and energy propagations through the detector. These simulations are used to both predict and search for expected physics within the real data.

My Goal:

- Incorporate astrophysics and nuclear interaction theory to simulate the WIMP velocities and interaction rates with the detectors
- Simulate weighted samples of events where we, on an event-by-event basis, calculate the 4-momentum of the nucleus after a WIMP-nucleus interaction
- Integrate this simulation into the standard CDMS analysis framework

Bottom line: We want simulations which produce samples that accurately reproduce the rate of collisions as a function the recoil energy

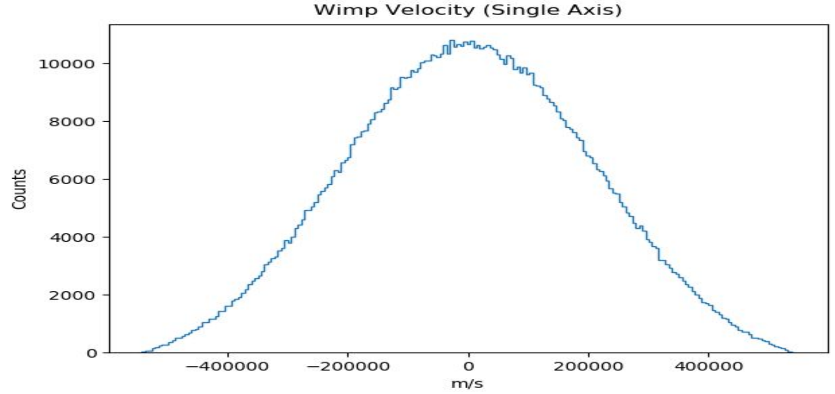
Simulating WIMPs

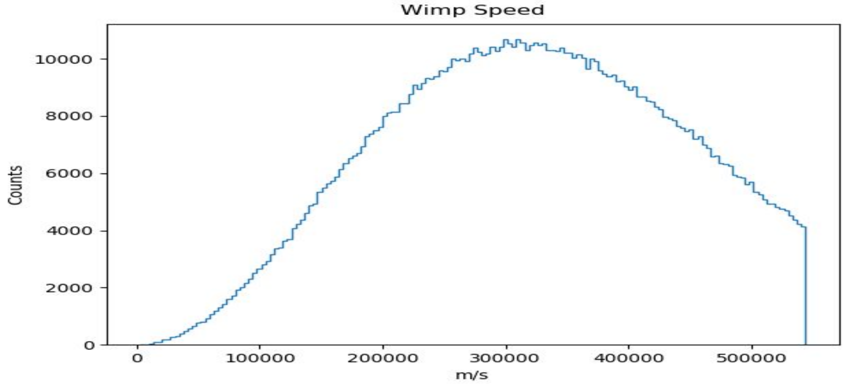
The simulation of WIMPs for use in SuperCDMS includes a calculation of the WIMP velocities in the Milky Way and then their interaction with a nucleus in the detector.

- Astrophysics: WIMP velocities are modeled to match distributions predicted for our galaxy
- Particle Physics: Each collision is weighted by a probability representing the relative chance of a WIMP scattering with a particular energy
- Kinematics: The recoil energies/momenta are then calculated dependent on the given WIMP mass, velocity, and a given atomic mass of recoiled nucleus

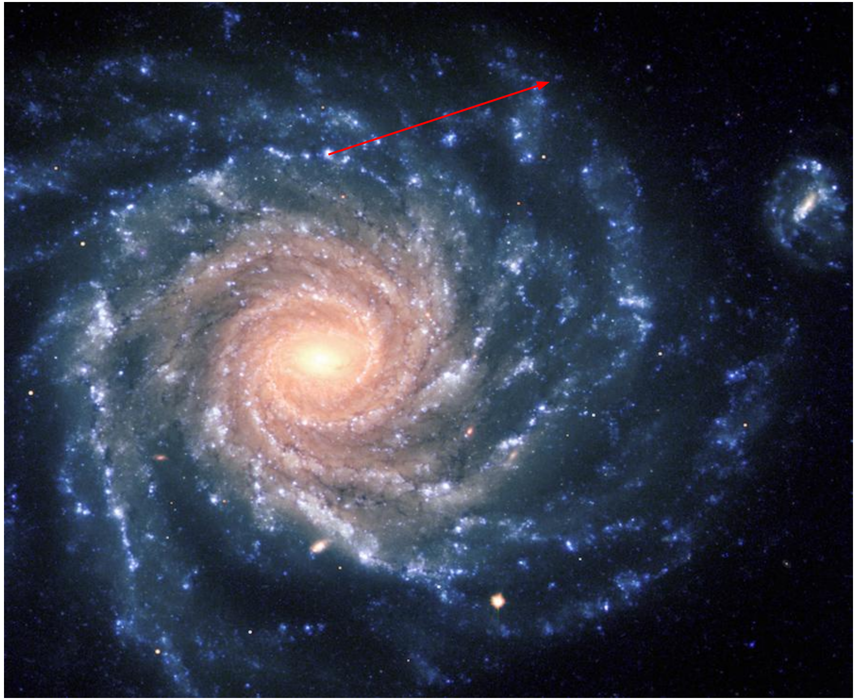
The crux of the calculation - For a set of masses and detector atom parameters, produce accurate recoil energies. For N events, produce the correct proportions of counts as a function of recoil energy.

Astrophysics: Velocity Distribution



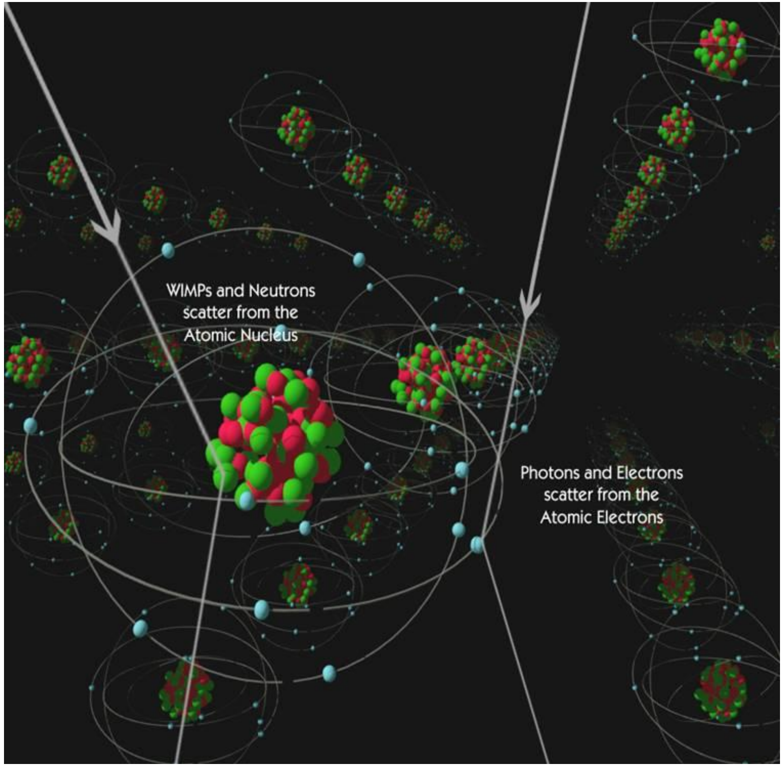


Wimp Escape Velocity: $v_{esc} = 544 \times 10^3$ m/s



- WIMPs are modeled to be a non-interacting gas at thermal equilibrium, hence a Maxwell–Boltzmann distribution
- We use the simplifying assumption that WIMPs are normally distributed throughout the galaxy, with a hard cutoff at the galactic escape velocity.

Particle Physics: Nuclear Parameters



Nucleus specific parameters such as the effective nuclear radius are needed to produce correct hit distributions.

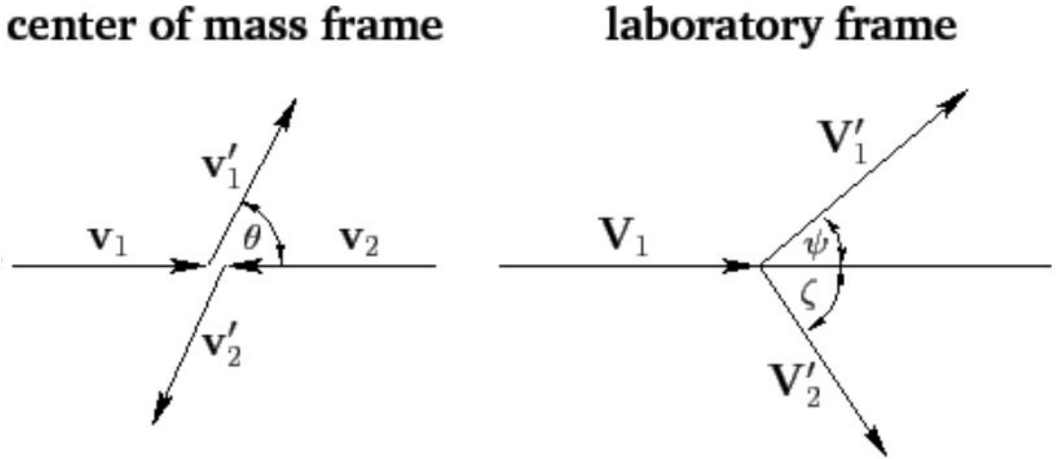
- A WIMP is more likely to interact with a larger nucleus.
- A weight is added to each collision as a WIMP is more likely to graze a nucleus rather than hit it head on.

Kinematics: Nuclear Recoil Energies

An elastic collision between a wimp and nucleus is the leading-order contribution to the energy transfer.

The experiment is chilled to a temperature of 40mK, making it safe to model the target nucleus as stationary compared to the WIMP.

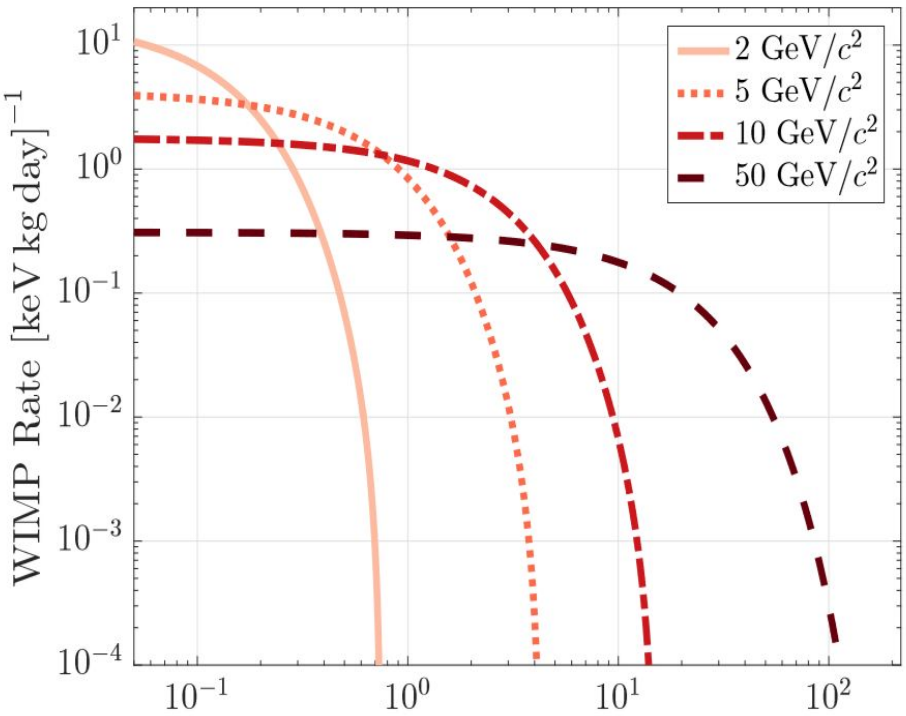
Given the two masses, WIMP velocity, and scattering angle, the energy and momentum transferred to the recoiled nucleus are given by:



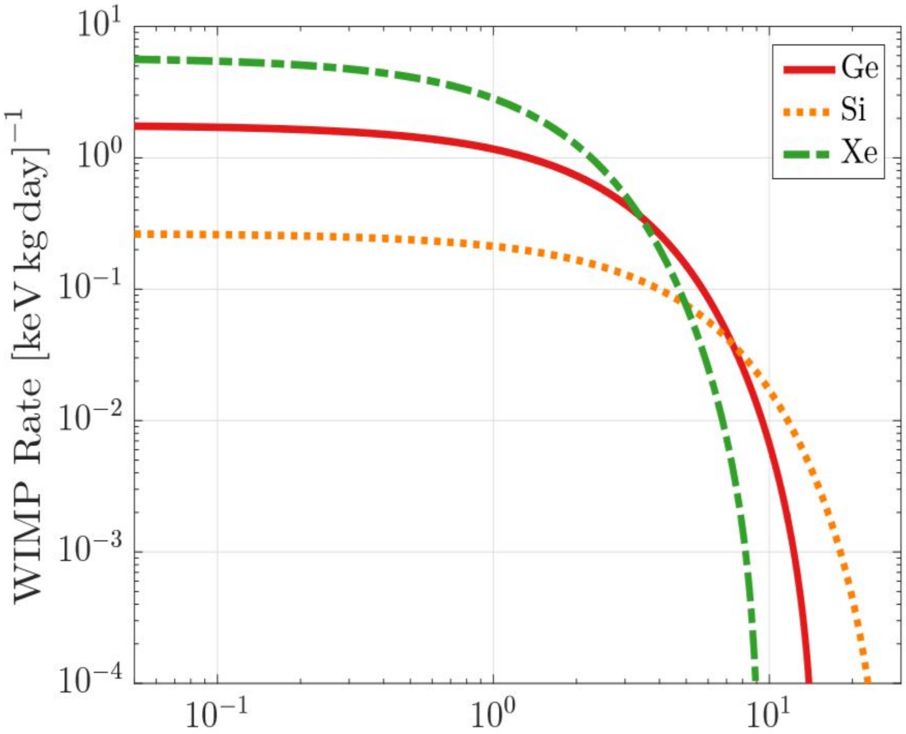
$$E_{recoil} = \frac{\mu^2}{m_N} V_1^2 [1 - cos(\theta)] \qquad p_{recoil} = \sqrt{2 m_N E_{recoil}}$$

Where μ is the reduced mass of the system: $\mu = \frac{m_W m_N}{m_W + m_N}$

Results from Previous Calculations as a Function of Wimp Mass and Detector Type



(a) Germainum Target



(b) 10 GeV/c^2 WIMP

Source: "Low-Mass Dark Matter Search Results and Radiogenic Backgrounds for the Cryogenic Dark Matter Search" by Mark Pepin; Figure 3.3 [4]

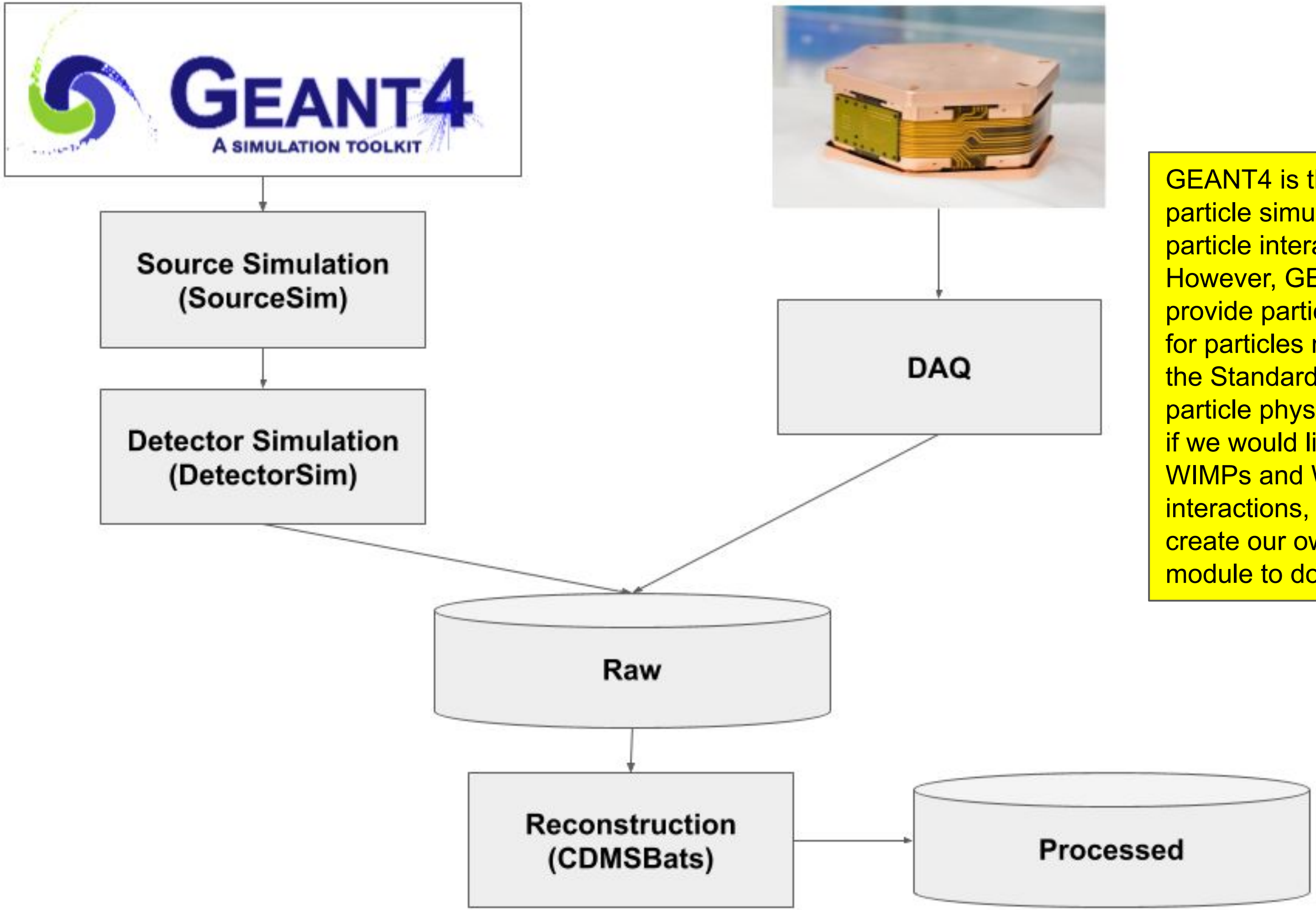
Goal for This Analysis

Show that our simulation creates samples of events where the distribution of recoil energies matches theory dependent on:

- Different masses of WIMPs
- Different detector materials, particularly germanium and silicon

Further, we would like to run these calculations through the CDMS Simulations Framework so we can analyze and study the results as if the hits were recorded by the real experiment.

The CDMS Simulations Framework and Reconstruction



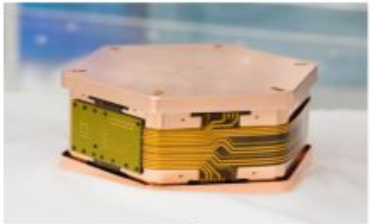
GEANT4 is the standard particle simulation tool for particle interactions. However, GEANT4 does not provide particle interactions for particles not contained in the Standard Model of particle physics. Therefore, if we would like to simulate WIMPs and WIMP interactions, we'll need to create our own custom module to do so.

GEANT4 — A SIMULATION TOOLKIT

WIMPSim

Source Simulation (SourceSim)

Detector Simulation (DetectorSim)



DAQ

Raw

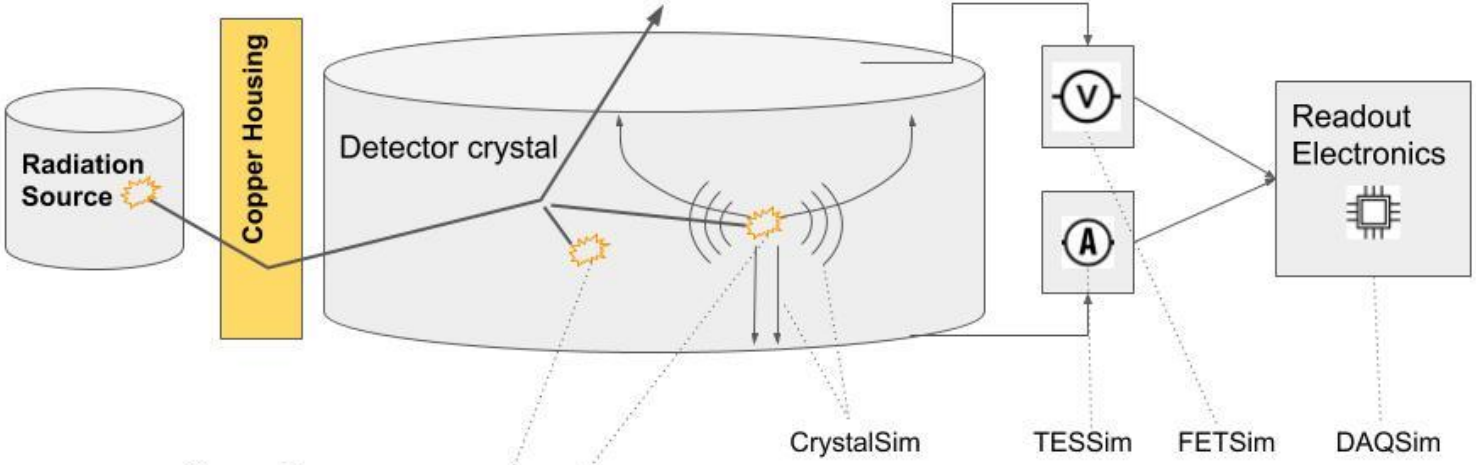
Reconstruction (CDMSBats)

Processed

GEANT4 is the standard particle simulation tool for particle interactions. However, GEANT4 does not provide particle interactions for particles not contained in the Standard Model of particle physics. Therefore, if we would like to simulate WIMPs and WIMP interactions, we'll need to create our own custom module to do so.

WIMPSim is added as a module, so WIMP energy deposits can now be simulated in the detector.

SuperCDMS Simulation



SourceSim
Simulation of particles that travel through things and eventually hit the detector or decay directly in the detector

Output: Recoil particle, energy and position in the detector

DetectorSim
Simulation of Crystal response to phonons and electron holes, the TES and FET as well as the DAQ

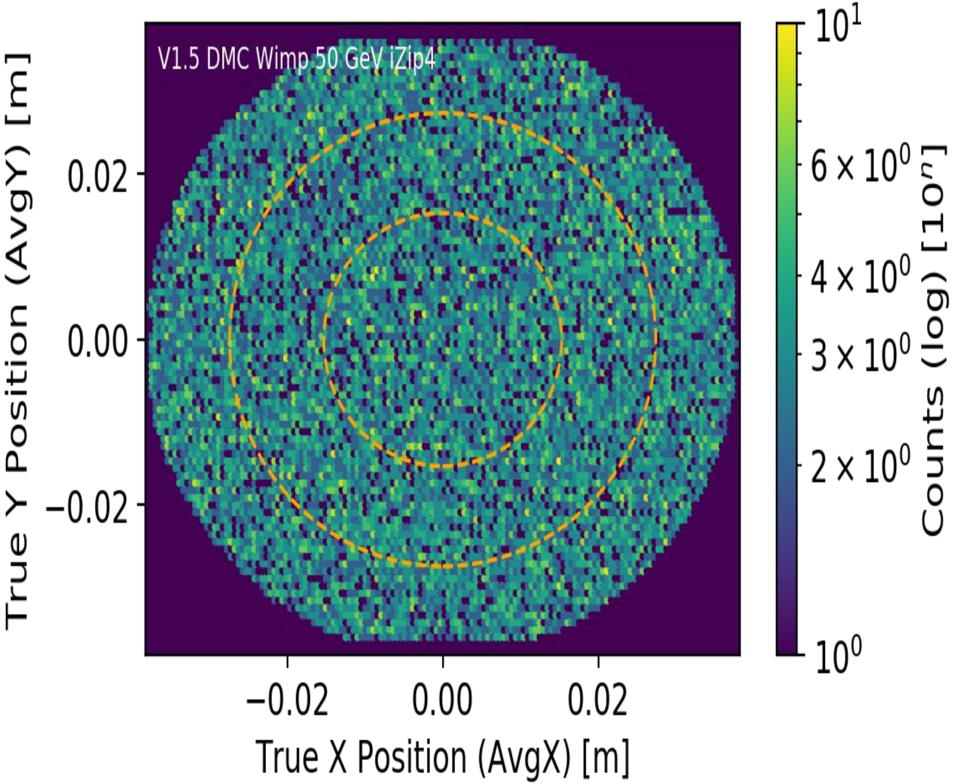
Output: Raw Data format

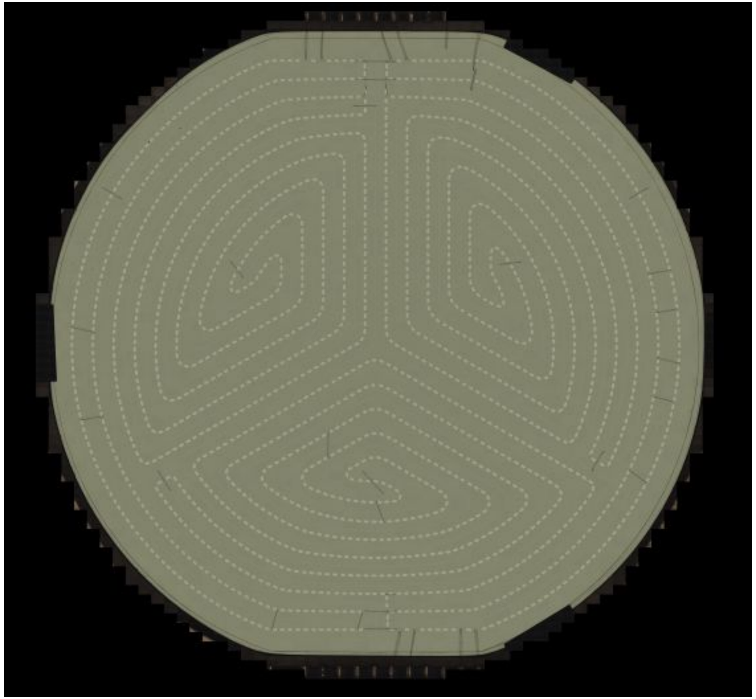
Simulation Results

Simulation Results

It works!

Simulation ran through a germanium detector from the CDMS Soudan experiment. Collisions are uniformly distributed throughout the detector. Here we show the location of 40k collisions for WIMPs at a mass of 50GeV.





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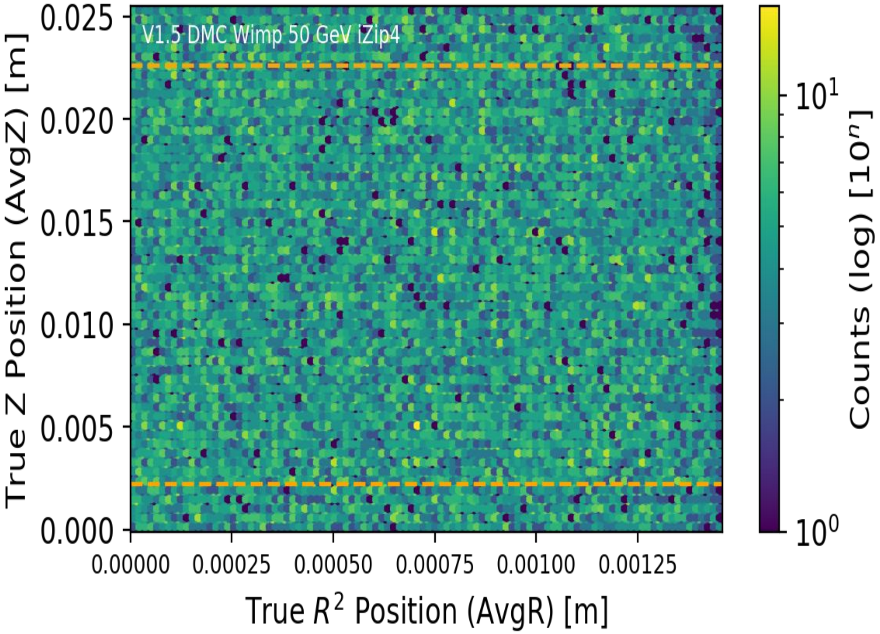
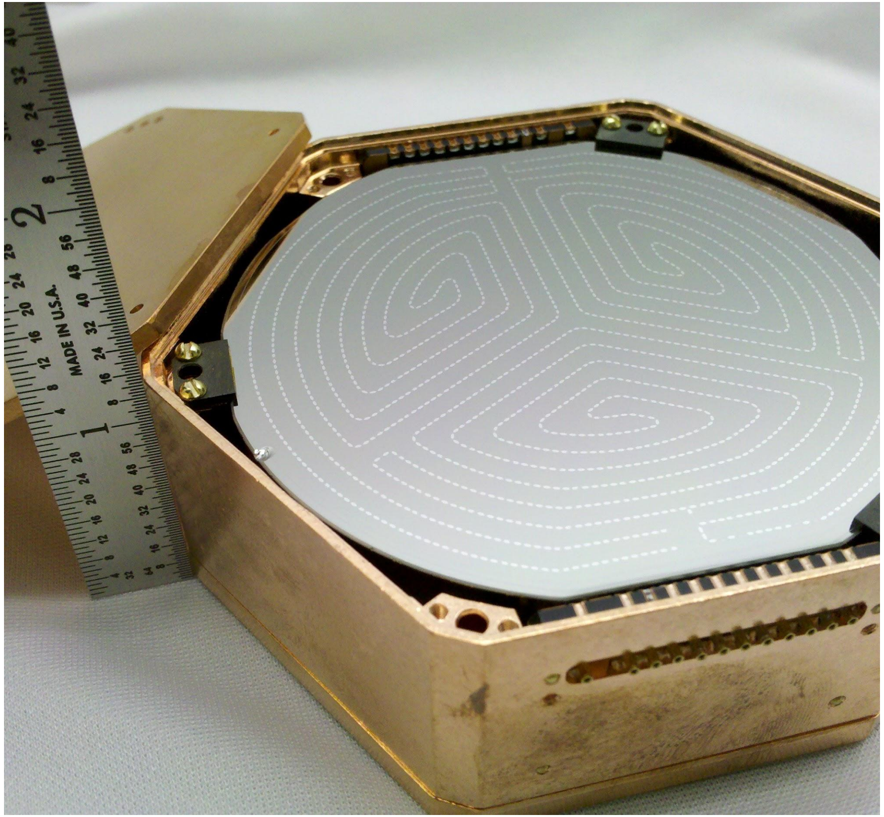
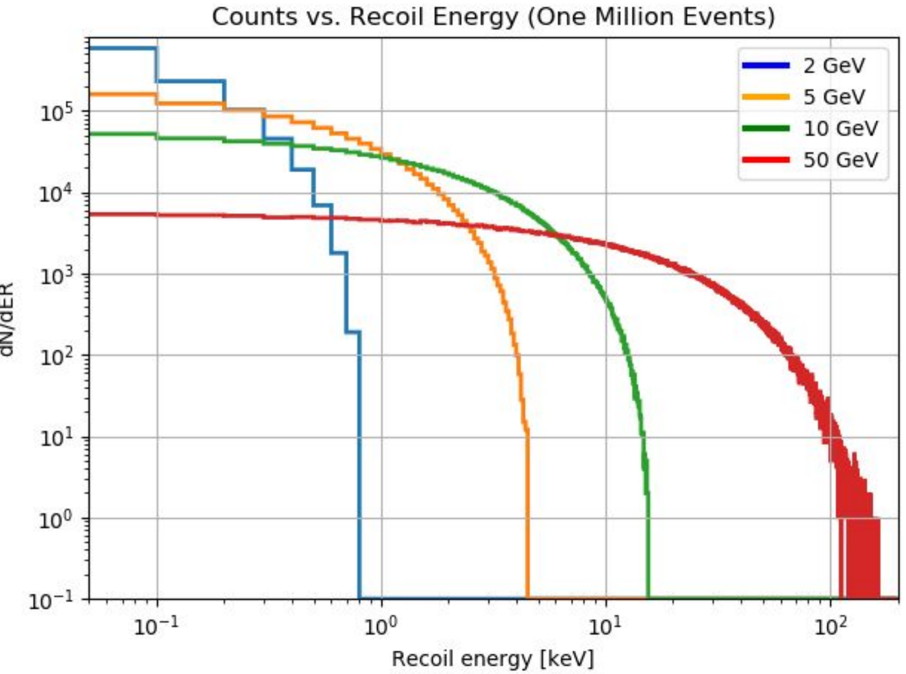
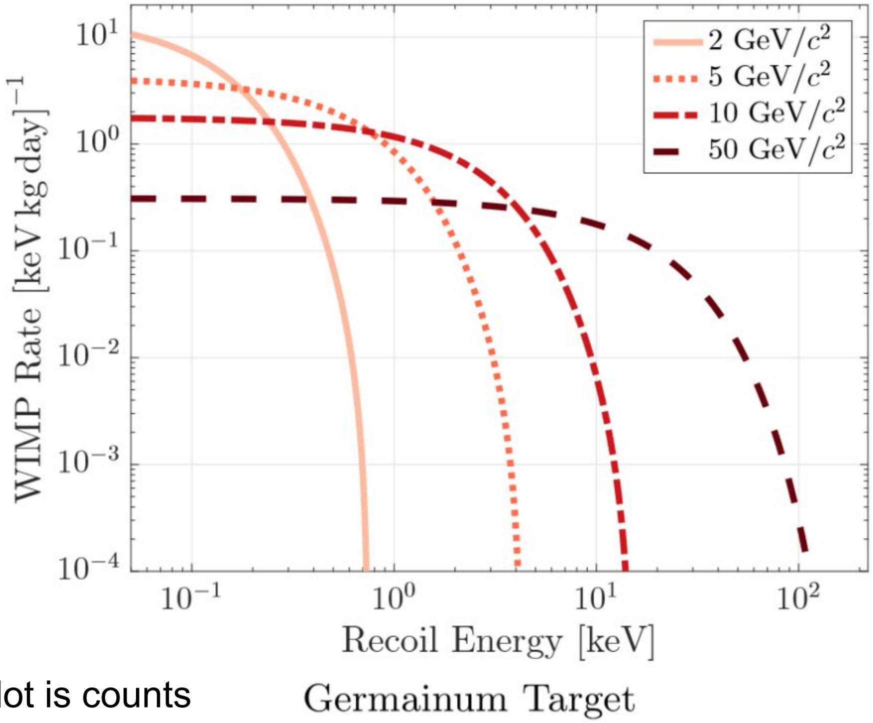




Simulation results; Counts vs Energy



Theory results; Rate vs Energy

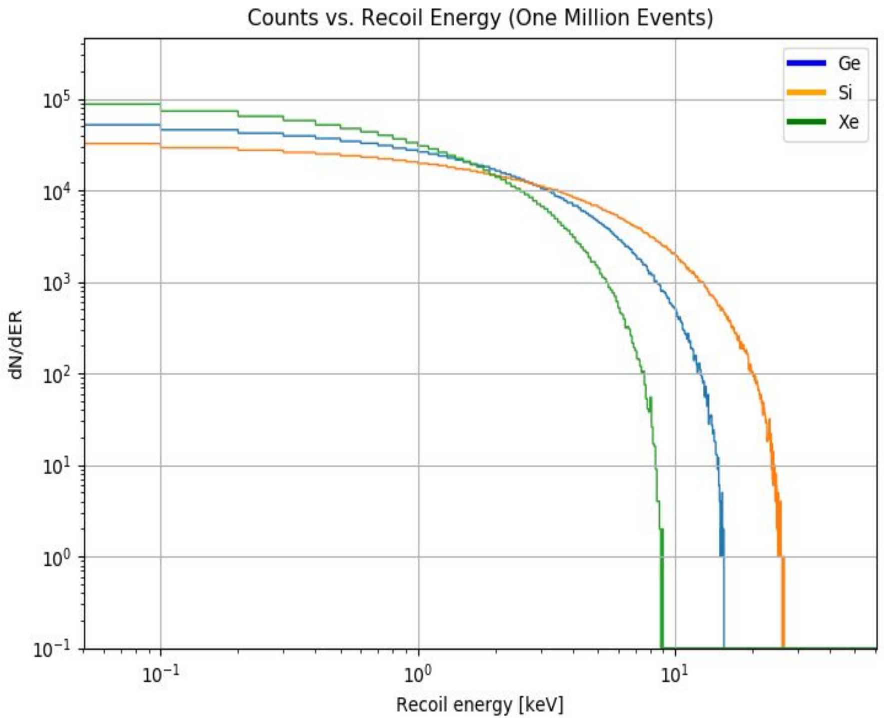


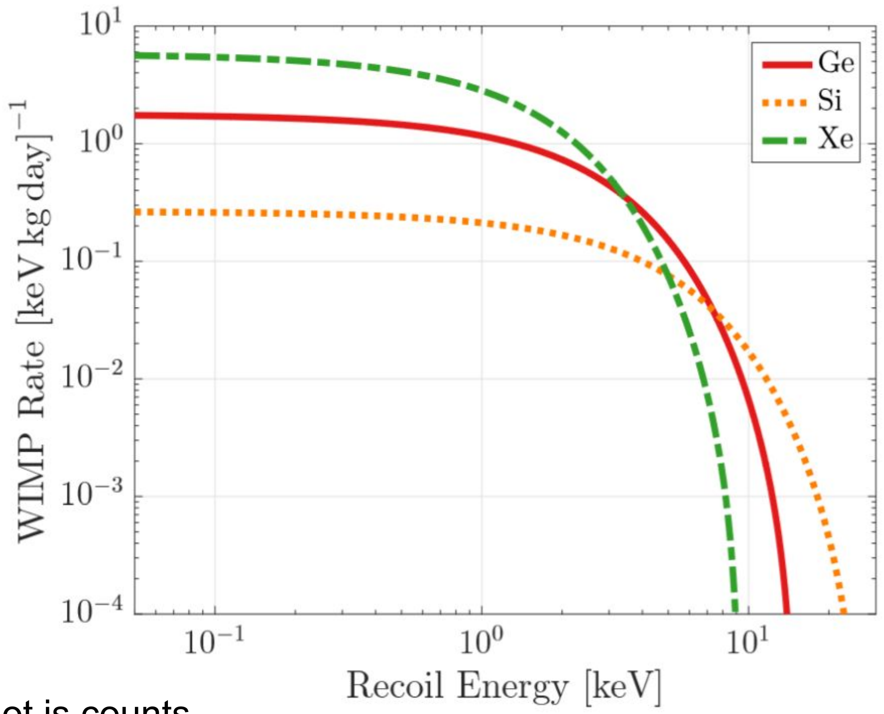
Mark Pepin; Figure 3.3 [4]

These are not directly comparable as the theoretical plot is counts per unit time, per unit mass of detector. Left hand side is a Monte-Carlo simulation of distributions at a large number of counts.

Simulation results; Counts vs Energy

Theory results; Rate vs Energy





$10\ \mathrm{GeV}/c^2$ WIMP

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Mark Pepin; Figure 3.3 [4]

What's Next/Work to Be Done

1. Additional quantitative validations and comparisons
2. Multiple types of WIMPs (Spin 0, Spin ½, Spin 1) allow for distinguishing between higher order collision terms in WIMP interactions
3. Adding hooks for annual modulation studies
4. Use this module to discover Dark Matter using the SuperCDMS Experiment at SNOLAB

Conclusion

We have used astrophysics, nuclear/particle physics and kinematics to create simulations of WIMPs with the SuperCDMS detector (WimpSim).

This program is very lightweight and does not have any dependencies on external libraries. Therefore it runs extremely quickly, and is very easy to modify, update, and analyze.

This model (WimpSim) provides an accurate description of:

- How much energy would be deposited in the detector for a given WIMP-Nuclei interaction
- The relative probability of such a reaction occurring

This model is fully integrated into the larger Standard CDMS Simulation framework, and is ready for analysis.

References

[1] Images credit: Van Albada et al. (L), A. Carati, via arXiv:1111.5793 (R). Observed velocities versus distance from the center of galaxy NGC 3198. The theoretical prediction before observations followed the trend labeled “disk”, but observations (black squares) showed constant, rather than decreasing velocity. Adding a contribution from a dark matter halo (center line) makes the theory match predictions.

[2] Review of mathematics, numerical factors, and corrections dark matter experiments based on elastic nuclear recoil [J.D. Lewin, RF. Smith]

[3] Model Independent Form Factors for Spin Independent Neutralino-Nucleon Scattering from Elastic Electron Scattering Data [Gintaras Duda, Ann Kemper, Paolo Gondolo]

[4] Low-Mass Dark Matter Search Results and Radiogenic Backgrounds for the Cryogenic Dark Matter Search [Mark David Pepin]

[5] The Inelastic Frontier: Discovering Dark Matter at High Recoil Energy [Joseph Bramante, Patrick J. Fox, Graham D. Kribs, Adam Martin]

Backup Slides

Parameters in WimpSim

Scattering angle (Θ): $-\pi/2$ to $\pi/2$

Velocity of WIMP in galactic frame: norm(0, 220x10^3 km s^{-1})†

Earth Phase: 0 - 2π

Nuclear skin thickness (s): 0.9e-15 m

Radius to half maxima of charge distribution for an atom (c): $1.23A^{-1/3}$ - 0.6e-15 m

Proton Nuclear Radius (a): 0.52e-15 m

†Drukier et al. argue that v_0 = V_r (the galactic rotation velocity) for a galaxy with a flat rotation curve.
Reported values for V_r, are: 243 ± 20 km s^{-1}; 222 ± 20 km s^{-1}; and 228 ± 19 km s^{-1}. I used V_0 = V_r = 230 km s^{-1}

Weights

Helm Form Factor:

$$F_{SI}(\Delta p) = 3\frac{j_1(\Delta p, r_n)}{\Delta p r_n} e^{-\frac{1}{2}(\Delta p s)^2}$$

$$r_n^2 = c^2 + \frac{7}{3}\pi^2 a^2 - 5s^2$$

$$c \simeq 1.23A^{1/3} - 0.60 \text{ fm}$$

$$s \simeq 0.9 \text{ fm}$$

$$a \simeq 0.52 \text{ fm}.$$

Wimp Escape Velocity: $v_{esc} = 544 \times 10^3$ m/s

WIMP recoil probability weighted by Helm form factor, and if velocity of wimp is greater than the escape velocity the weight is set to 0

Theory Portion

What we expect a count rate vs eV distribution to look like:

$$\frac{\mathrm{d}R}{\mathrm{d}E_{\mathrm{r}}} = \underbrace{\frac{N_T m_T}{2 m_\chi \mu_T^2}}_{\text{Detector}} \cdot \underbrace{\left[\sigma_0^{\mathrm{SI}} F_{\mathrm{SI}}^2(E_{\mathrm{r}}) + \sigma_0^{\mathrm{SD}} F_{\mathrm{SD}}^2(E_{\mathrm{r}})\right]}_{\text{Particle and Nuclear}} \cdot \underbrace{\rho_0 \int_{v_{\mathrm{min}}}^{v_{\mathrm{max}}} \frac{1}{k} \frac{f(\boldsymbol{v}, \boldsymbol{v}_E)}{v} \, \mathrm{d}^3\boldsymbol{v}}_{\text{Astro}}$$

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Focus on spin independent WIMPs

Why is there a minimum velocity in $\frac{\mathrm{d}R}{\mathrm{d}E_{\mathrm{r}}}$?

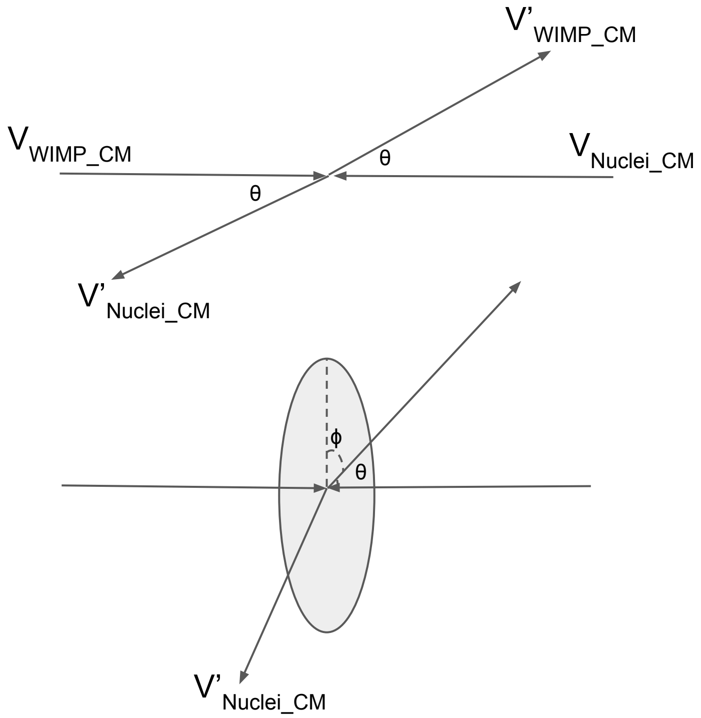
Since recoil energy is given by: $\Delta E = \frac{\mu^2}{m_2} V_1^2 (1 - cos(\theta))$

There is only a specific range of wimp energies (and hence velocities) that will produce a given recoil energy

$$v_{\min} = \sqrt{\frac{m_T E_{\mathrm{r}}}{2\mu_T^2}}$$

Calculating Recoil 3-Momentum

Let's define a coordinate system where all of the WIMP's momentum is in the x-direction. For simplicity's sake, let's also assume the recoil will always occur in the x-y plane. In the CM frame of this coordinate system:



Calculating 3-momentum is a 5 step process.

1. Rotate to WIMP coordinate system
2. Boost to CM frame of reference
3. Calculate recoil momentum 3-vector using a given θ and ϕ

- We now have a recoiled nucleus momentum 3-vector in the WIMP's coordinate system, in the center of mass frame of reference.

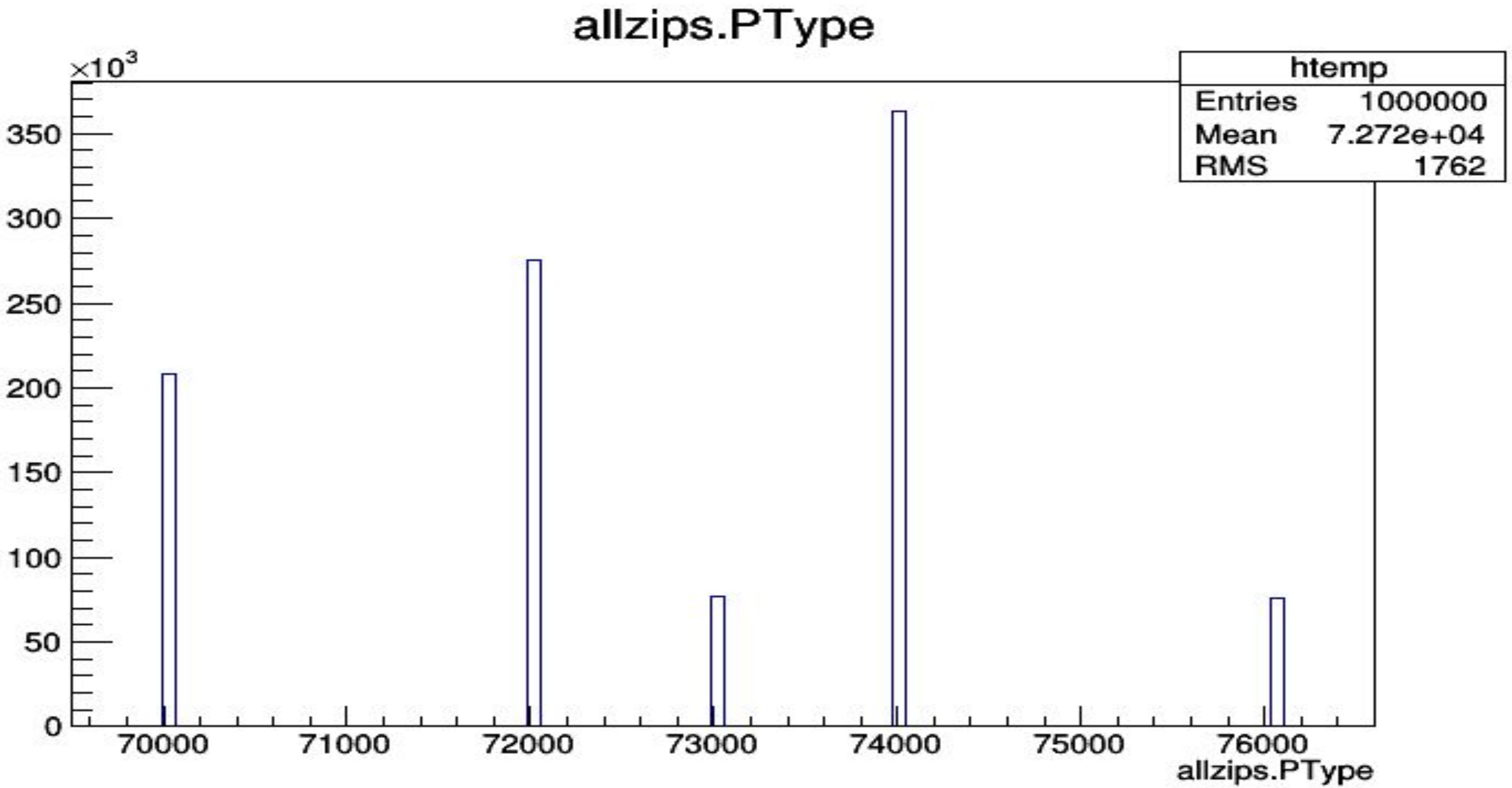
4. Boost back to the detector frame of reference
5. Inverse the rotation to rotate back into the detector coordinate system

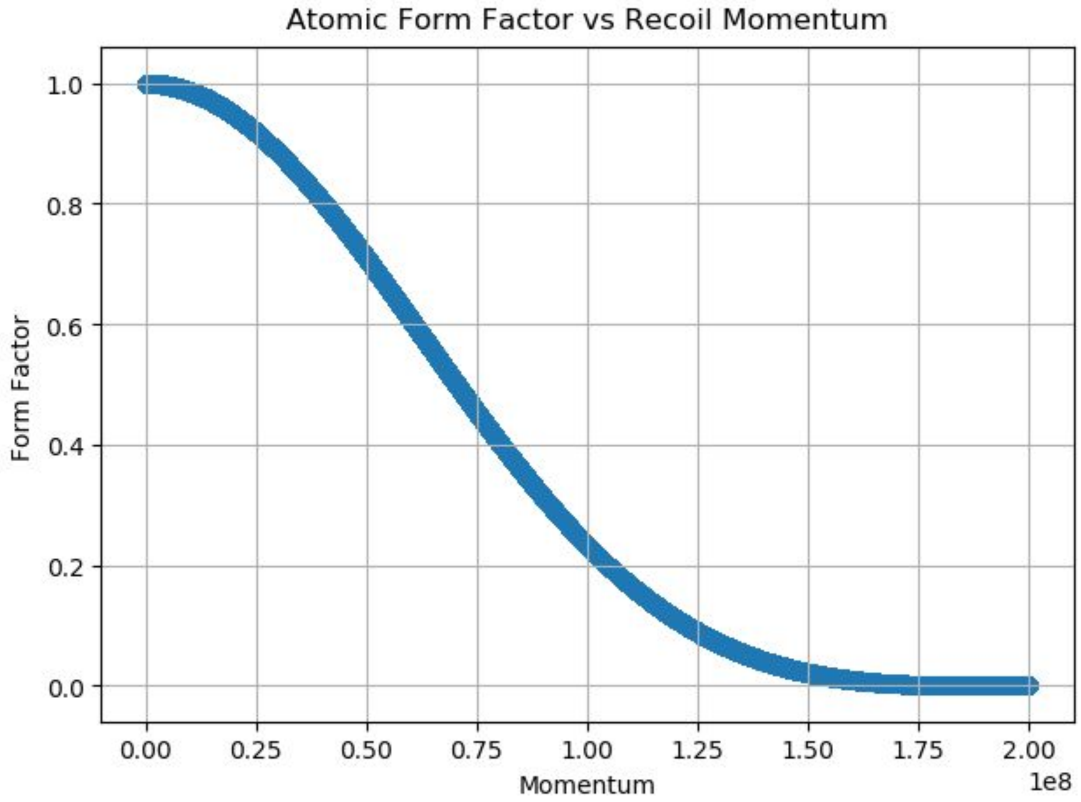
Given a unit-axis of rotation **k** and an angle of rotation θ, we can rotate an arbitrary vector **v** using Rodrigues' rotation formula.

$$\mathbf{v}_{\mathrm{rot}} = \mathbf{v}\cos\theta + (\mathbf{k}\times\mathbf{v})\sin\theta + \mathbf{k}\,(\mathbf{k}\cdot\mathbf{v})(1-\cos\theta)\,.$$

To rotate back, simply replace θ → -θ

Counts vs Nuclei Type - For izip14, these are all isotopes of Germanium (1e6 hits)

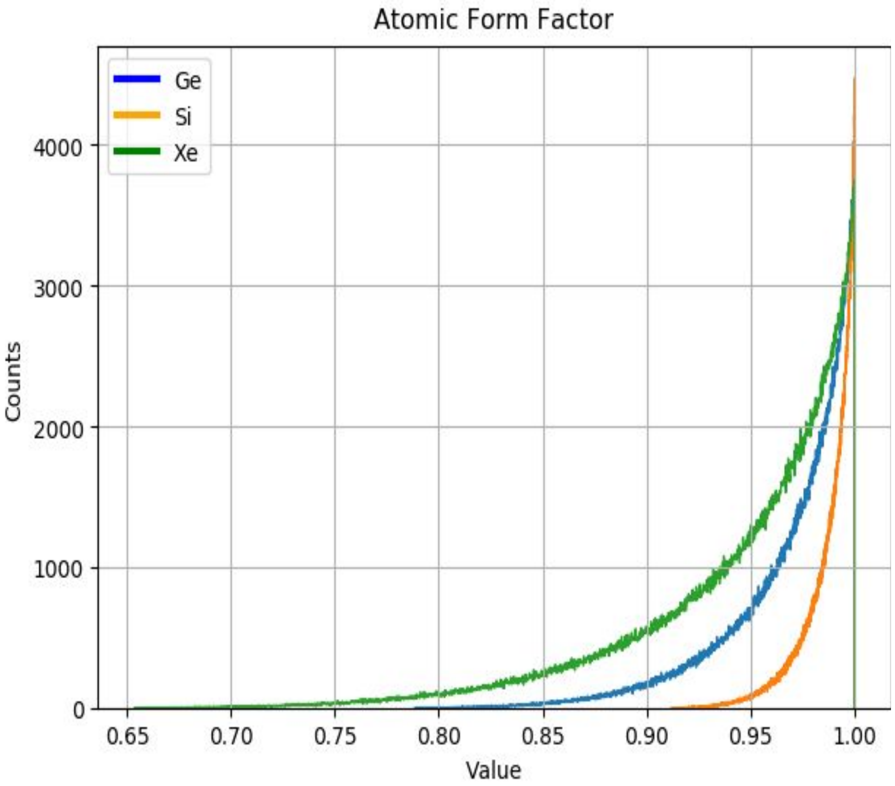


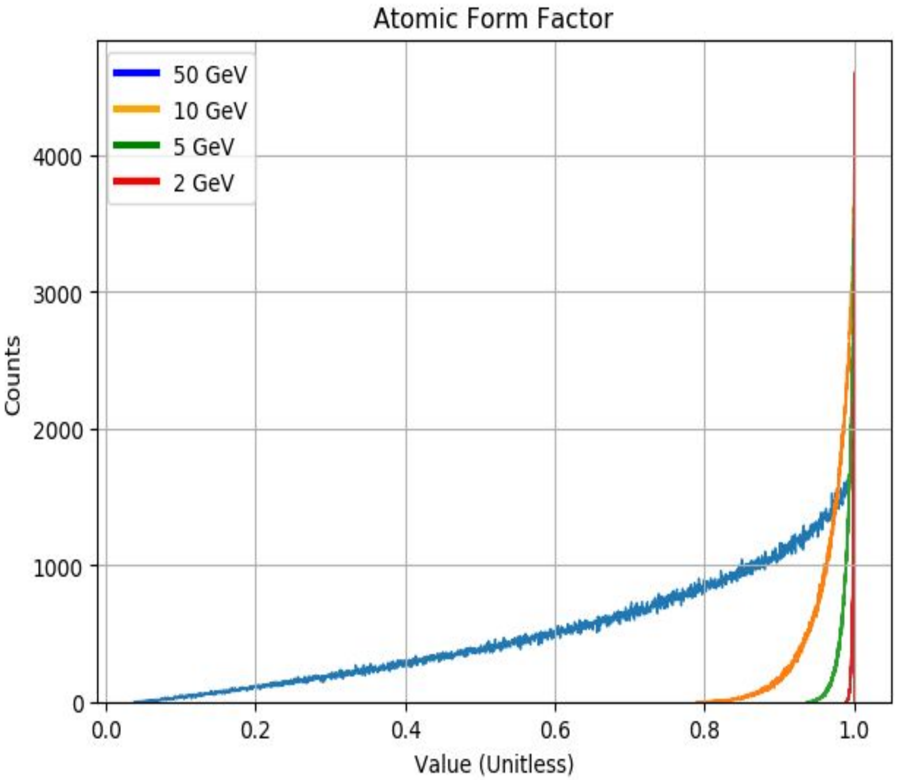


The atomic form factor depends on the recoil energy/momentum. The higher the relative recoil momentum of a particular collision, the less likely it is to occur.

This graph is specifically the form factor for a Germanium isotope - atomic weight A = 70

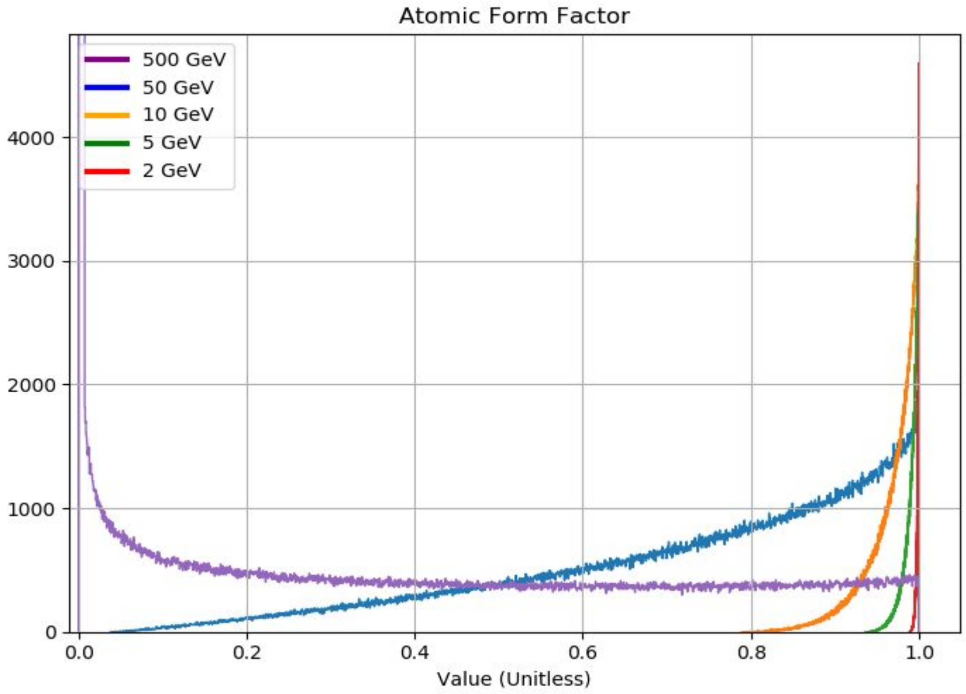
Squaring the Form Factor gives the probability of a particular collision occurring.



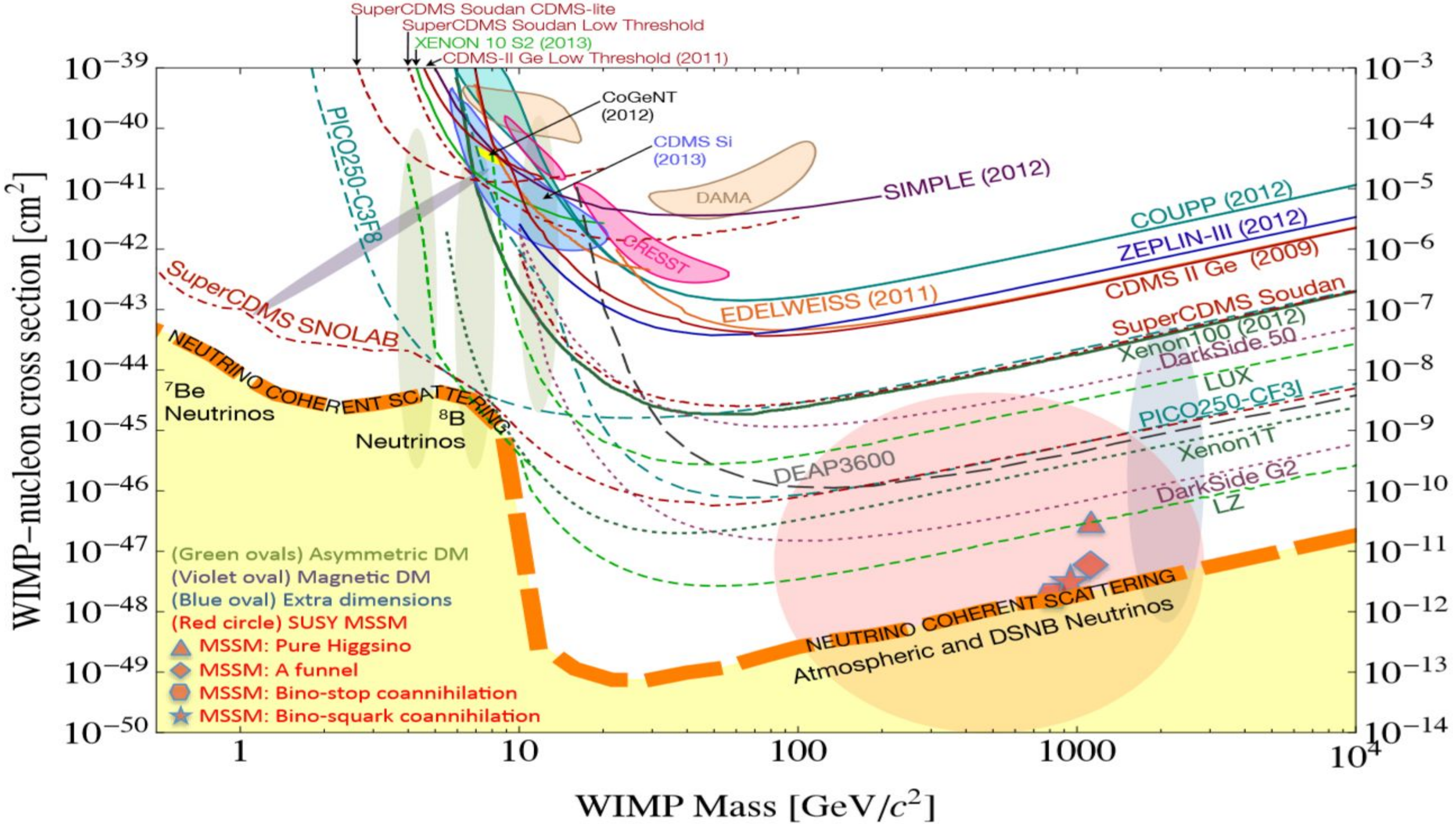


Collision counts per type (1e6 runs)

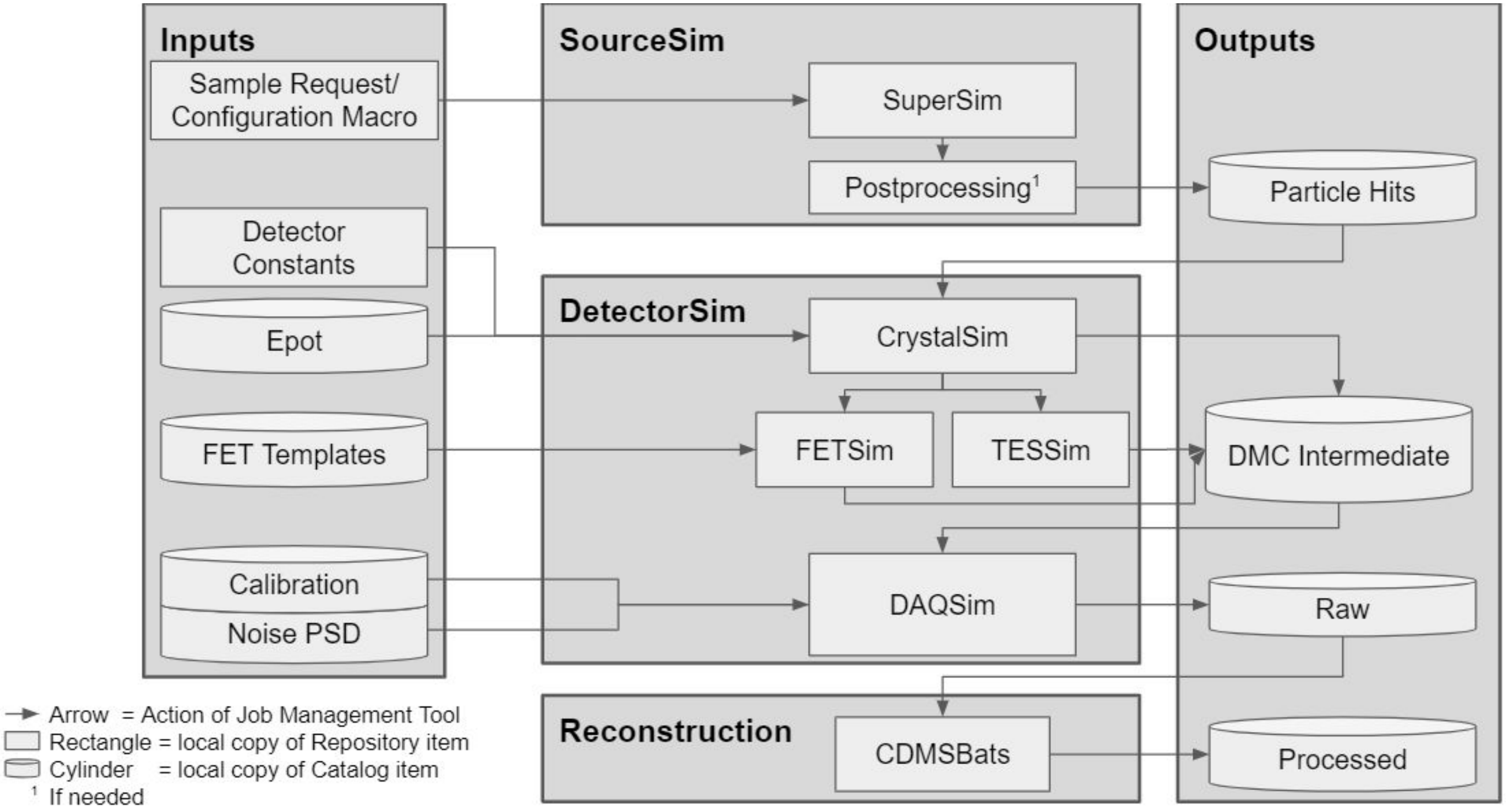
WIMP Mass; Target	Events	% Calc
2 GeV; Germanium	998365	99.8%
5 GeV; Germanium	990646	99.1%
10 GeV; Germanium	967952	96.8%
10 GeV; Silicon	987017	98.7%
10 GeV; Xenon	944313	94.4%
50 GeV; Germanium	733118	73.3%
500 GeV; Germanium	391052	39.1%



The CDMS Experiment



Source Simulations and Data Analysis for CDMS



Source Simulations and Data Analysis for CDMS