

Using simulations to make a sensitivity projection of the electroweak mixing angle measurement in a DUNE Near Detector environment

Preliminary Exam

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The Standard Model of particle physics and the electroweak interaction

The Standard Model (SM) of particle physics describes the nature and behavior of all known fundamental particles in the universe as well as the forces that they use to interact.

This project will be focused on the electroweak force, which is a combination - or unification - of the electromagnetic force and the weak force and centers around the transfer of mass and charge in interactions involving neutrinos by way of W and Z bosons.

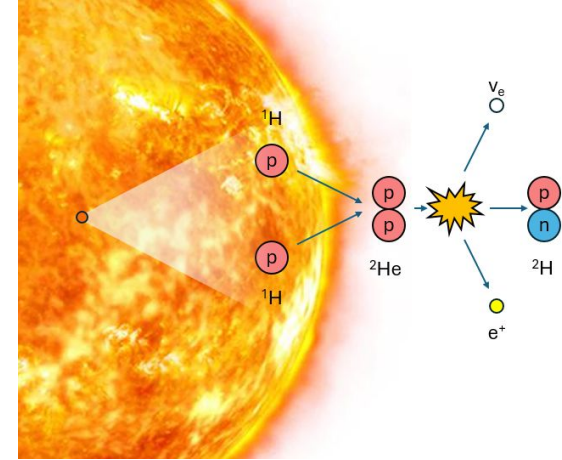
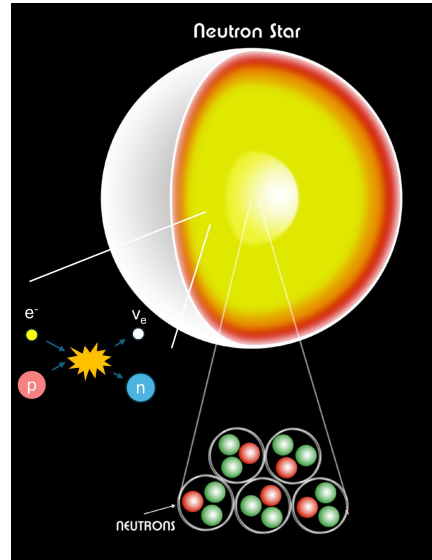
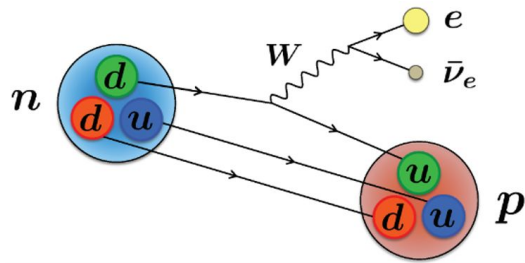
Standard Model of Elementary Particles

three generations of matter (fermions)			interactions / force carriers (bosons)		
	I	II	III		
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 124.97 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
	u up	c charm	t top	g gluon	H higgs
	d down	s strange	b bottom	γ photon	
	e electron	μ muon	τ tau	Z Z boson	
	$< 1.0 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$	$\approx 91.19 \text{ GeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$
	0	0	0	0	± 1
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

QUARKS
LEPTONS
GAUGE BOSONS VECTOR BOSONS
SCALAR BOSONS

Common occurrences of electroweak physics

The electroweak force is involved any time nucleons decay making it the main force involved in nuclear interactions.



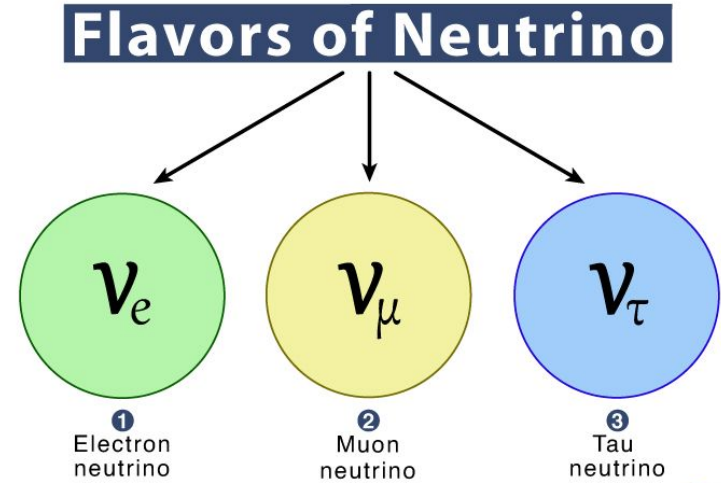
The electroweak force is also seen in the fusion of hydrogen to fuel a main sequence star, as well as in the electron capture process used to change protons to neutrons in the extreme conditions of a neutron star.

Neutrinos

Neutrinos are fundamental particles described in the SM and are known for:

- Being the most abundant particle in the universe
- Having three “flavors” or generations - aligning with the electron, muon, and tau leptons
- Traveling very fast - very close to the speed of light
- Having a spin value of $\frac{1}{2}$ - classifying them as “fermions”
- Having no color charge - meaning they don’t interact via the strong force, classifying them as “leptons”
- Having very small mass - meaning they can interact via gravity, but negligibly
- Being electrically neutral - meaning they don’t interact via electromagnetism
- Having an extremely small cross section - meaning they barely interact at all

Because of these characteristics, neutrinos essentially only interact with other particles via gravity, but primarily via the weak force.



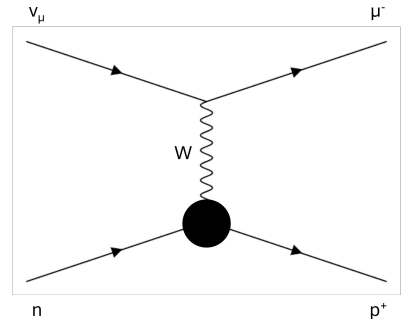
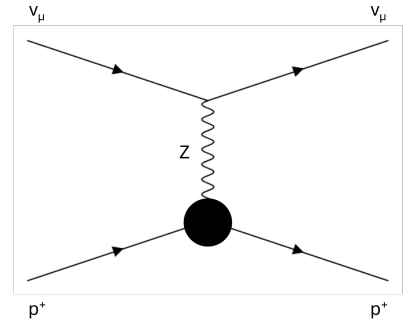
W^\pm and Z^0 bosons

The electroweak force is propagated by two types of bosons

W bosons: These bosons are electrically charged and transfer charge from one particle in an interaction to another, and changing the types of involved particles.

Z bosons: These particles are electrically neutral and act as heavy photons, acting in scattering interactions where the particle types remain the same.

Both of these bosons are massive and only act over short distances.



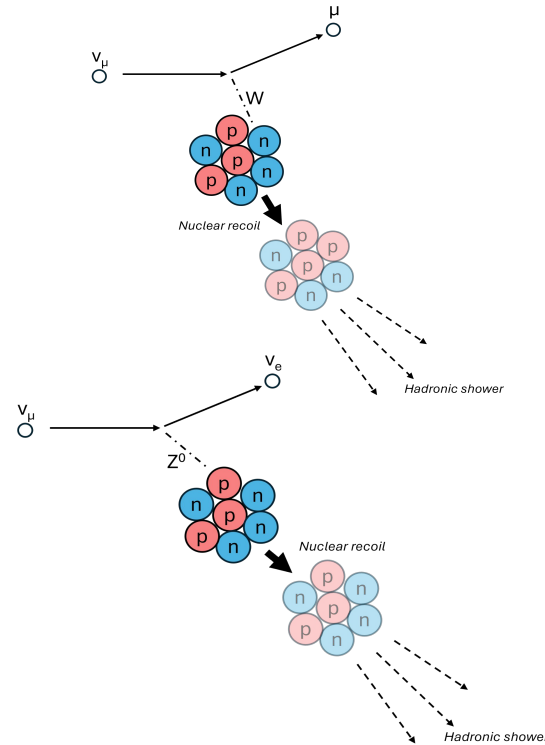
The weak mixing angle in neutrino interactions: Charge or Neutral Current

Because there are 2 types of bosons (W and Z), an electroweak interaction can occur in one of two different ways:

- 1) A W boson passes from one particle to another. Electric charge is transferred from one particle to another (therefore called Charge Current Interactions).
- 2) A Z boson passes from one particle to another. No electric charge is exchanged and only information is transferred between charged particles (therefore called Neutral Current interactions).

The rate at which particles interact via one process or the other is determined by the weak mixing angle (θ_W or sometimes $\sin^2\theta_W$).

The SM, which describes all known properties of and interactions between particles sets the weak mixing angle, however any additional interactions (a.k.a. “new physics”) that aren’t accounted for would affect the value of the mixing angle making it deviate from the theoretical value in the SM.

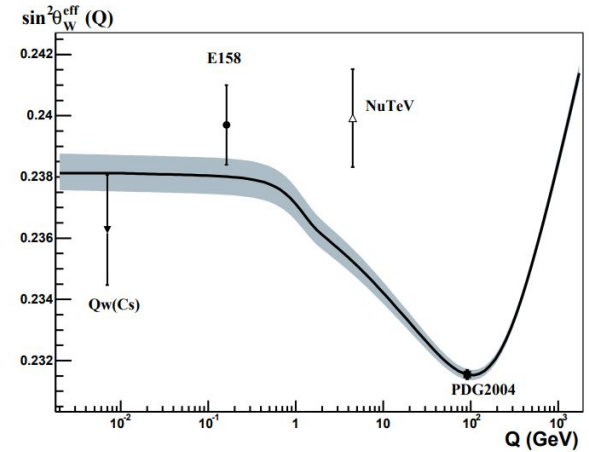


Motivation: Necessity for an improved weak mixing angle measurement

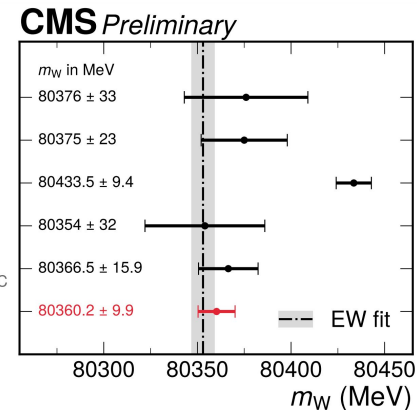
The predictions of the Standard Model include all the physics we are aware of. Deviations from the SM expectations could indicate new physics. Therefore, measuring θ_W with fidelity and small uncertainties is imperative so as not to miss new physics or suggest new physics that isn't there.

Recently multiple experiments have measured θ_W with a high enough precision to be statistically significant. NuTeV (see *upper right*) and CDF II (see *lower right*) published results that suggest conflicting results from many of the currently accepted expectations.

It becomes necessary to measure θ_W with high precision to follow up on these potential clues.



LEP combination
Phys. Rep. 532 (2013) 119
D0
PRL 108 (2012) 151804
CDF
Science 376 (2022) 6589
LHCb
JHEP 01 (2022) 036
ATLAS
arxiv:2403.15085, subm. to EPJC
CMS
This Work



The Deep Underground Neutrino Experiment (DUNE)

DUNE is the flagship international neutrino experiment based in the United States. It is part of the Long Baseline Neutrino Experiment and spans from FermiLab in Chicago, IL to Sanford Underground Research Facility in Lead, SD.

Currently DUNE is planned and scheduled to go online in the early 2030s, and is expected to provide cutting-edge insights into the fundamental nature of matter and the universe.

There are already plans for upgrades in the future to improve performance, including charge pixels to replace detector readout electronics. *We will go into more detail in the next section.*



Overview of approach

Because DUNE is not scheduled to go online for many years, this project will take a phenomenological approach instead.

- Develop simulations to model particle events and detector response with the new charge pixel technology
- Develop event reconstruction software for analyzing the data from the simulated upgraded detector
- Project the sensitivity of the electroweak mixing angle in a simulated detector
- Project the dependence the sensitivity will have on detector efficiency

Outline

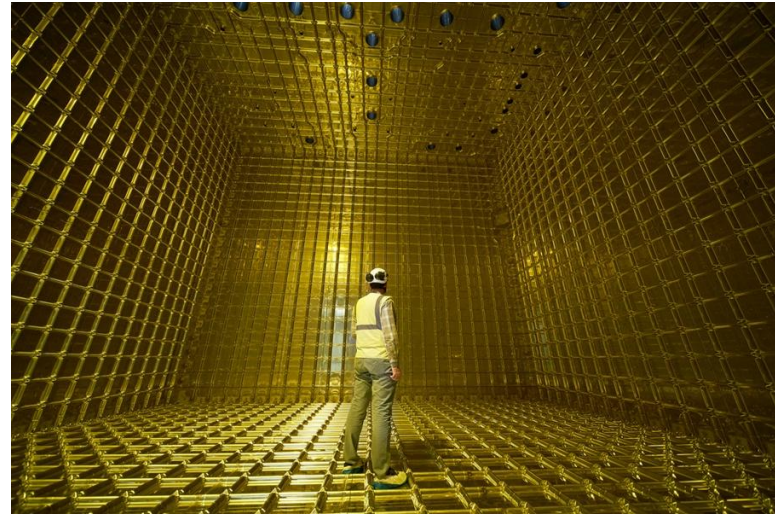
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Neutrino experiment design

Neutrino experiments have had to face the challenge of trying to detect extremely elusive particles. The common goal of these experiments has been to maximize the chance of “catching” a neutrino in the measurable volume.

Maximizing the chances of observing a neutrino interaction can be done in three ways:

- **Make the target bigger.** Neutrino detectors are being built larger and more efficient every year with the goal to improve the sensitivity of the measurements in order to capture as much data as possible and reduce the run times necessary for experiments.
- **Increase the number of chances.** Neutrino experiments have used brighter and brighter neutrino sources, usually taking the form of a neutrino beam produced with an accelerator. Using a beam of neutrinos, like the one produced at the PiP-II beam at FermiLab, allows experiments to reach the statistics necessary to make a measurement with a small uncertainty.
- **When all else fails, wait longer.** Neutrino experiments can run over the span of years to produce the amount of data necessary to make a measurement.

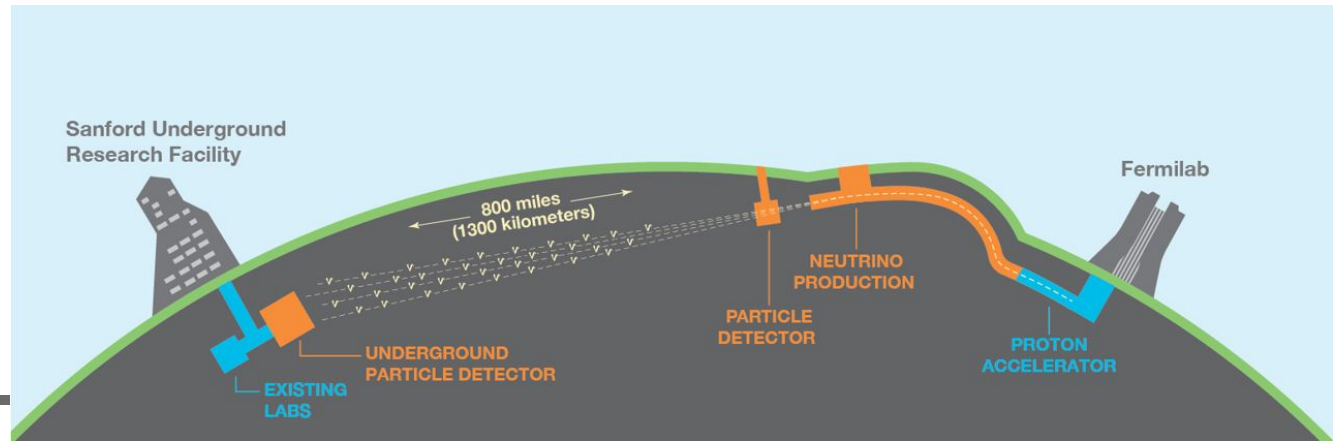


The Deep Underground Neutrino Experiment (DUNE)

DUNE consists of 2 cutting edge Liquid Argon Time Projection Chamber (LArTPC) neutrino detectors:

- DUNE Near Detector (DUNE ND) - a high resolution, modular detector right at the source
- DUNE Far Detector (DUNE FD) - a large 4-module, 40kton detector 800 miles away and almost a mile underground.

Both detectors will be placed along the path of PiP-II, the brightest muon neutrino beam in the world, and are designed to capture neutrino events in four dimensions.



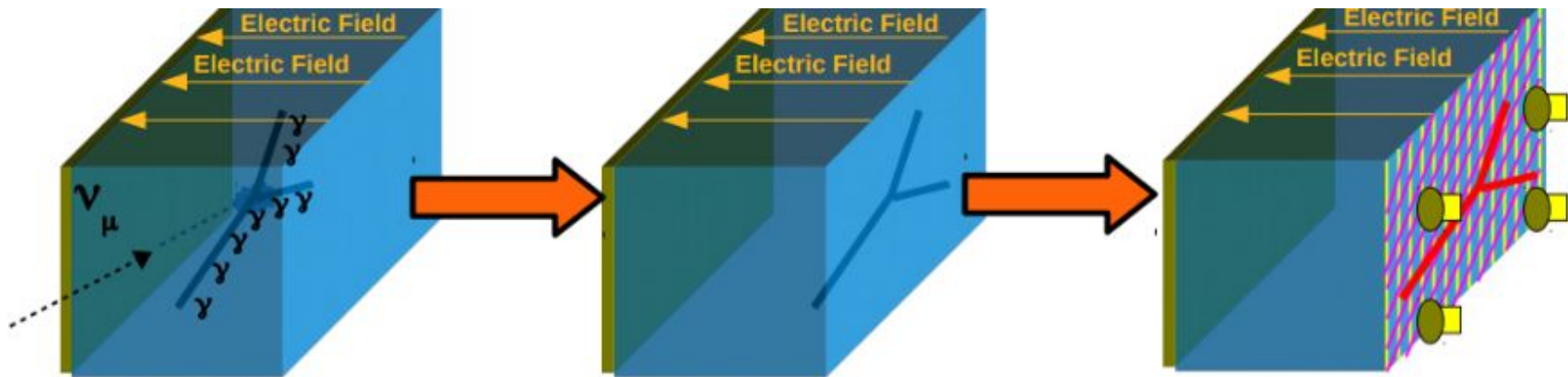
A hypothetical LArTPC detector

A Liquid Argon Time Projection Chamber (LArTPC) is a type of detector that makes 4D measurements of particle events.

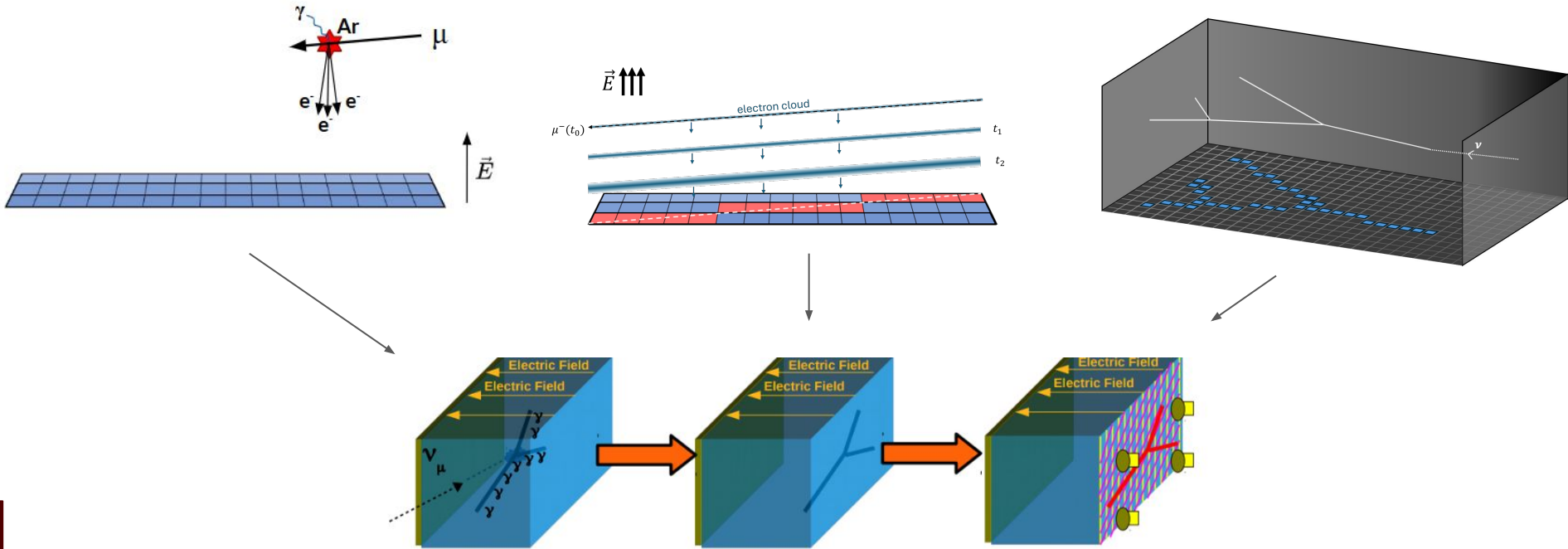
Each LArTPC is a giant tank of liquid argon, chosen for its favorable properties for the detection of particles, in a constant electric field, with a detector wall on one side.

When charged particles traverse the volume, for example as part of the final state products of a neutrino interaction, they will cause the LAr to scintillate (emit light) and will knock free electrons along its path through ionization. These free electrons will be pulled in the electric field towards the detector wall where the detector technology and readout electronics will turn the physical electrons into a signal and data that can be read out and processed.

Once the data is collected and processed, the full particle event can be reconstructed in space and time.



A deeper pictorial look inside a LArTPC

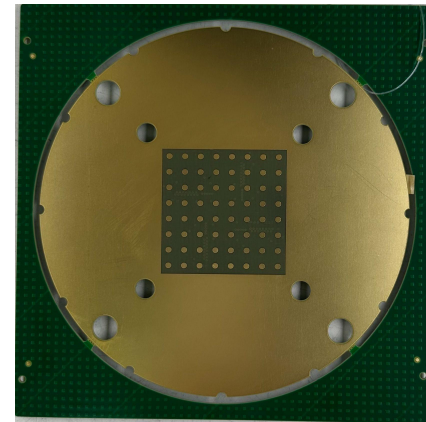
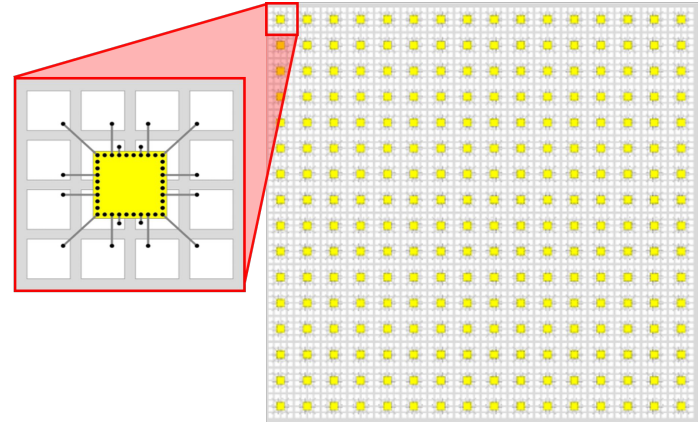


Charge Pixels (QPix) to replace wire technology

DUNE is currently slated to use wire planes as the detector wall. Requiring 1.5 million wires which are constantly reading out with GPS precision (100MHz), DUNE is expected to produce up to in excess of 1EB of data every year. Since neutrinos interact infrequently, most of this data is 0's and is useless.

QPix is a charge pixel designed with the concept of least action in mind. While being inherently 2-dimensional will help with complex topologies in particle events, these pixels will be programmed to only read out if there is something to be read out. In other words, QPix will remove all of the useless 0's that will flood the wire plane readout system.

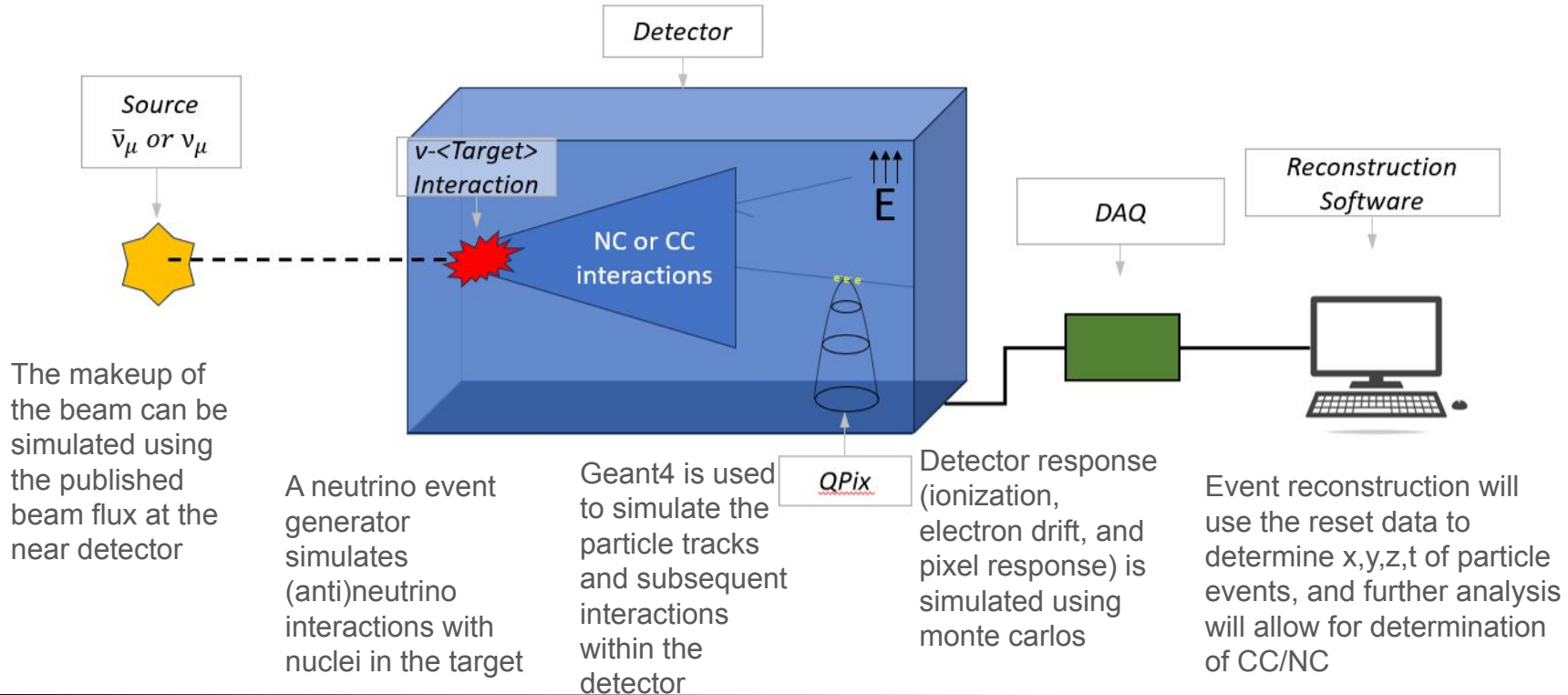
The location of the pixel allows us to get the X and Y position, while the time of arrival allows us to get the Z position for relativistic particles



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Overview



Simulating the source and nu-N interactions in the detector volume

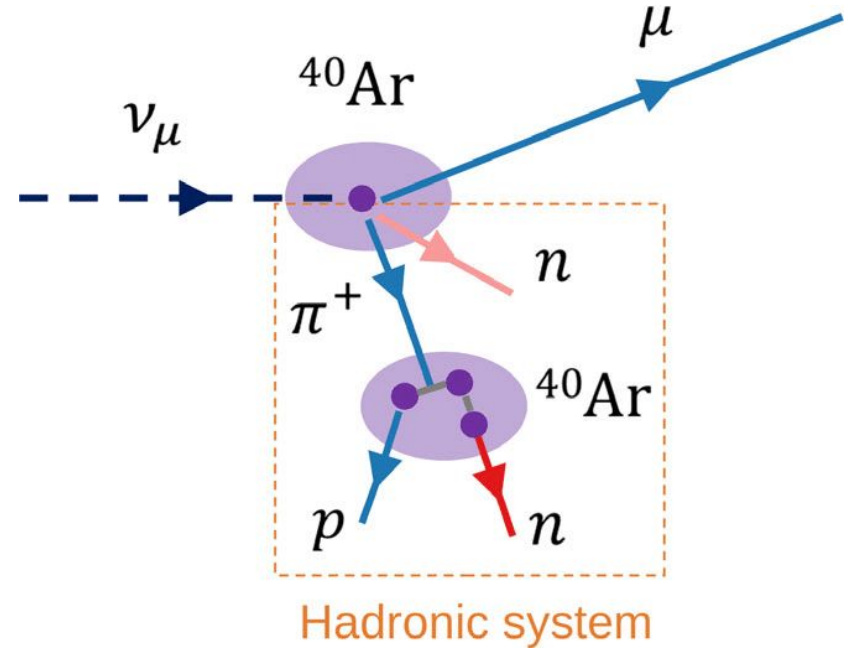
GENIE is the neutrino event generator used for the QPix simulation.

The beam flux at DUNE ND has been specified in the DUNE TDR. These fluxes can all be loaded into GENIE's event generator with a flux file to simulate this specific beam.

Using cross sections, it runs monte carlos to simulate the possible interactions between a neutrino from the beam and a target.

It will simulate the initial interaction and step out the interactions that would occur within the nucleon and determine the stable final state particles that would exit the nucleus.

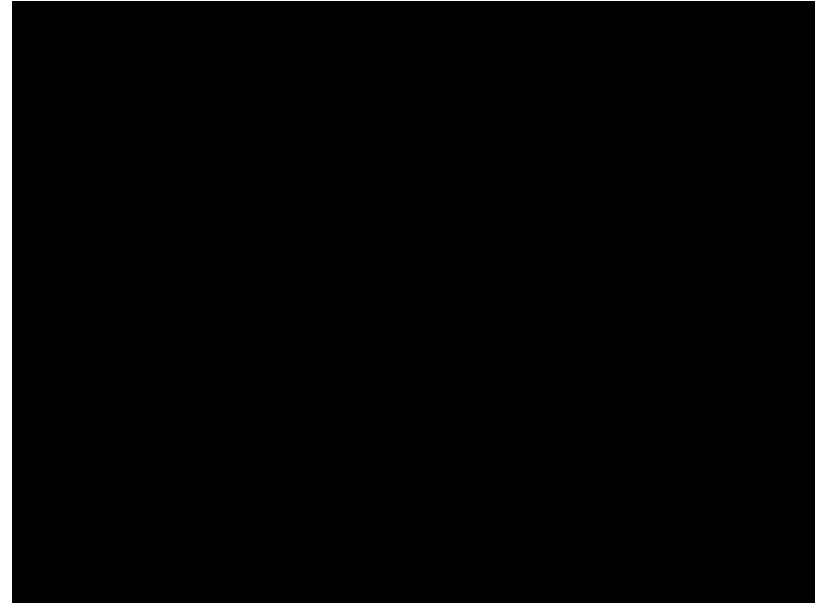
These final state particles will be fed into Geant4 in the next stage as initial generator particles.



Simulating detector response

The stable final state particles are fed into qpixg4 as initial particles and an event (including all energy deposits) is simulated using Monte Carlo simulations.

The energy deposit information is then used in qpixrtd to simulate the ionized electrons along the particle trajectories. This software package simulates constant drift in the electric field and Gaussian diffusion in 3 dimensions, and maps the final electron locations into a simulated pixel array.

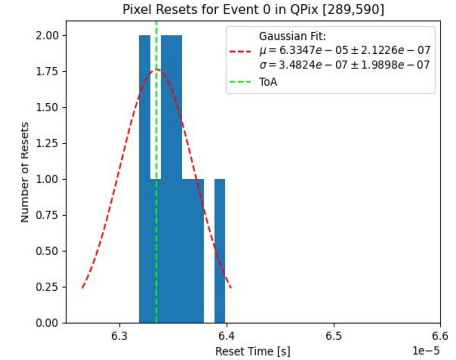


Simulating event reconstruction

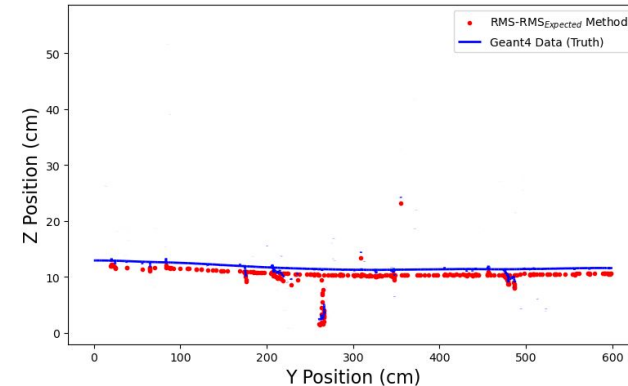
Using the pixel data produced by qpixrtd, it is then possible to reconstruct the particle event.

At this point in time, we have been able to fit the distribution of resets in each pixel to a Gaussian and measure the mean and rms. These values are used to calculate drift distance (z) of the original energy deposit, while the location of each individual pixel will give the location (x,y) of the deposit.

Using these methods, we can project the z resolution of a QPix-equipped DUNE-like detector.



Q-Pix Reconstruction of YZ Positions
Event 0



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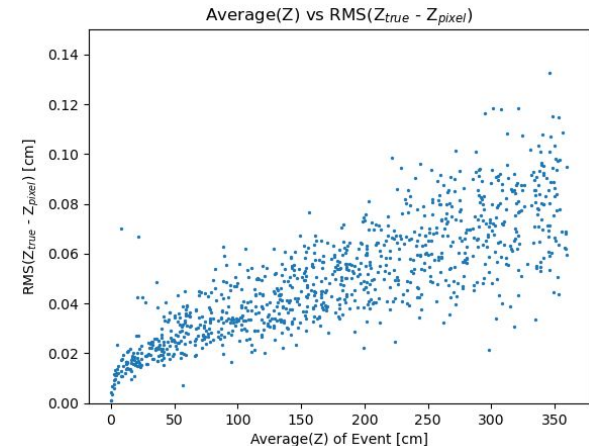
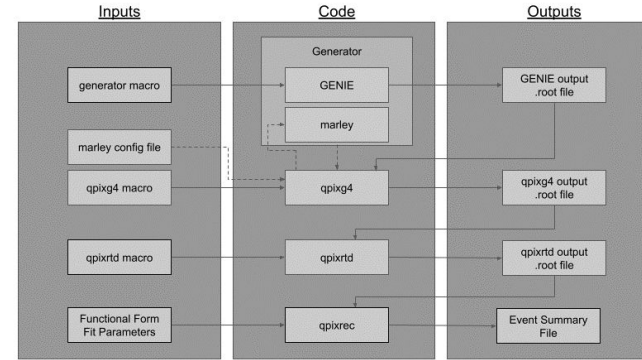
Status of QPix software packages

The QPix software packages have been developed to be able to successfully simulate all parts of the source, interaction, detector response and event reconstruction.

The latest version of the reconstruction software has been tested and found to have a z-dependent accuracy that asymptotes to 1.32cm at $z=0\text{cm}$ and 26.25cm at $z=180\text{cm}$. The precision of the software asymptotes to 0.17mm at $z=0\text{cm}$ and 0.5mm at $z=180\text{cm}$.

The high precision allows the physics to be conserved in reconstruction, while the accuracy just results in an overall translation in Z of the entire event.

Q-Pix: Input, Code, and Output Structure



Calculating the weak mixing angle from charge current and neutral current interaction counts

Using these simulations, we will be able to make a sensitivity projection of the electroweak mixing angle measurement in a DUNE ND environment.

The weak mixing angle can be calculated from the number of CC and NC events in an isoscalar (equal number of protons and neutrons) target using the Paschos-Wolfenstein relation. Using this same relation it is possible to propagate uncertainties to determine the statistical uncertainty on a given measurement from a data taking run.

$$R^{\nu(\bar{\nu})} \equiv \frac{\sigma(\overset{(-)}{\nu} N \rightarrow \overset{(-)}{\nu} X)}{\sigma(\overset{(-)}{\nu} N \rightarrow \ell^{-(+)} X)} = (g_L^2 + r^{(-1)} g_R^2)$$

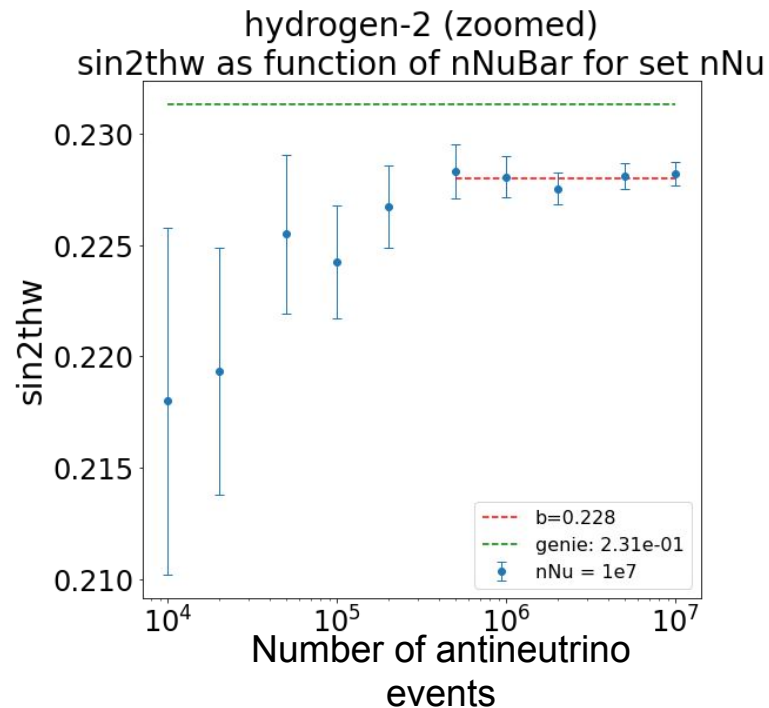
$$R^- = \frac{R^\nu - \tau R^{\bar{\nu}}}{1 - \tau} = \frac{1}{2} - \sin^2 \theta_W$$

$$r \equiv \frac{\sigma(\bar{\nu}_\mu N \rightarrow \mu^+ X)}{\sigma(\nu_\mu N \rightarrow \mu^- X)} \sim \frac{1}{2}$$

Hydrogen-2: Applying the Paschos-Wolfenstein relation to an ideal case

The Paschos-Wolfenstein relation (PW) relates the ratios of CC and NC events occurring in an isoscalar (equal number of protons and neutrons) target to the value of the weak mixing angle ($\sin^2\theta_W$). This measurement can be repeated for different numbers of events in samples to project a measurement of the weak mixing angle.

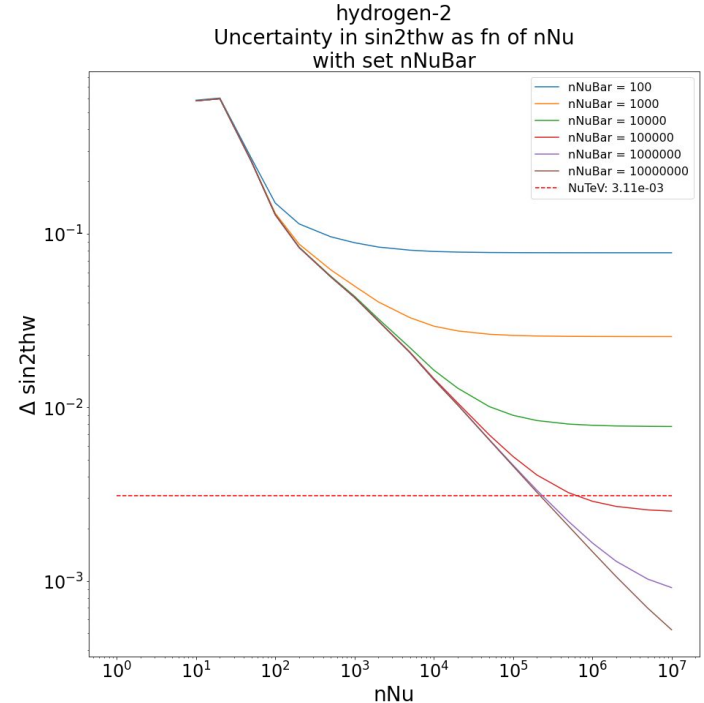
In the plot to the right, this method was used with hydrogen-2, an isoscalar target, for different amounts of data collection as a test. The green line shows the value of $\sin^2\theta_W$ used in GENIE to create the sample, while the red line shows the asymptote for the calculated value of $\sin^2\theta_W$. This plot shows that even with an isoscalar target, some form of correction factor is needed. The same can be done with LAr, using a separate correction factor that would include correction for the non-isoscalar nature of argon.



Hydrogen-2: Projecting uncertainties

By using the PW relationship and the counts of CC and NC events, not only can we calculate $\sin^2\theta_W$ but we can also calculate the uncertainty of the measurement. By assuming that a measurement on the weak mixing angle will be dominated by statistical uncertainties, and propagating the errors through the PW relation, we can project the sensitivity of the measurement for nEvents of neutrino and antineutrino beams separately.

The plot on the right shows a preliminary relation between the uncertainty on a $\sin^2\theta_W$ measurement and the number of neutrino events for set numbers of antineutrino events in a isoscalar hydrogen-2 target. The dashed red line shows the NuTeV sensitivity. This plot shows that the asymptotes are determined by the number of antineutrinos. It assumes 100% efficiency



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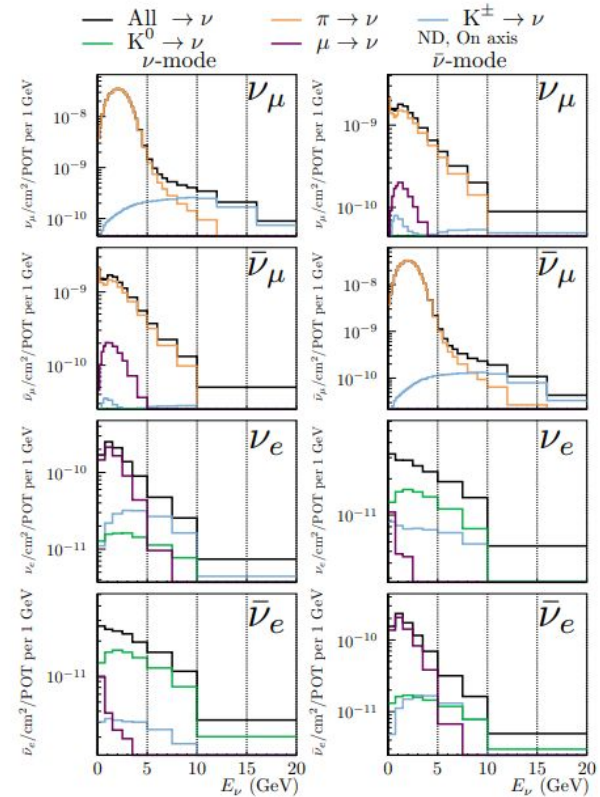
Applying the Paschos-Wolfenstein relation to a liquid argon target

Now that we know the PW relation works with an isoscalar target, the next step is to apply this to the isovector liquid argon target. We expect there to be a similar behavior as seen in the hydrogen-2, where the measurement will asymptote to a value as nEvents increases, but we expect the deviation from the truth value to be much larger.

A correction factor will be applied to the calculation to account for the isovector nature of LAr.

Incorporating Background

The PW relation works best with muon neutrino and antineutrino events. In fact, any electron neutrino events is expected to cause problems in the calculation as the charge current events are expected to look indistinguishable from the neutral current events. As such it is important to understand the flux of the beam for each flavor of neutrino to accurately account for these events in the calculations.

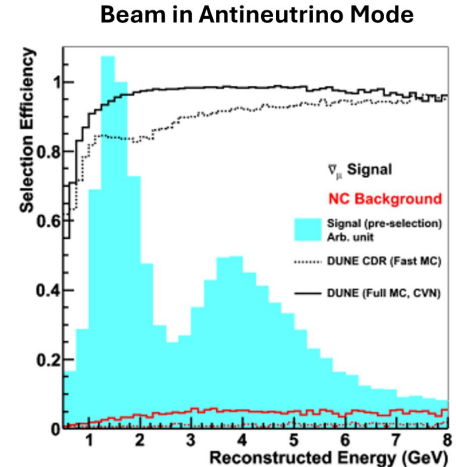
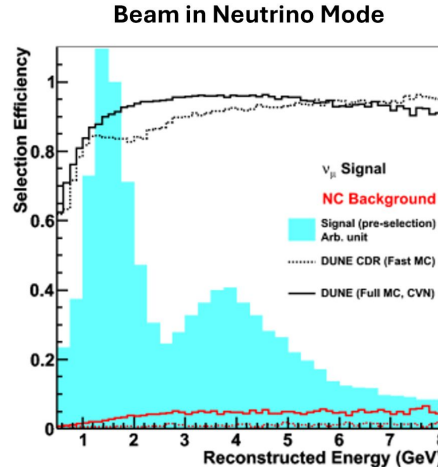


Determination of Selection Efficiencies

Previous simulations based on the wire-plane technology have estimated the selection efficiency (the efficiency of successfully labeling a CC event) of wire-plane-equipped DUNE to be about 85% across the expected energy distribution. This is far different than the selection efficiencies of previous experiments which sat optimistically at ~40%.

It is a reasonable assumption that the actual selection efficiency of DUNE (wires) will be different than this, and even more reasonable to assume that equipping DUNE with QPix will change this more.

For the scope of this project, we will pick an assortment of selection efficiencies (spanning the entire range, but more densely picked around the 85% mark) and explore how this will impact DUNE's sensitivity to the weak mixing angle.



Protons on Target (PoT) and Years of Data (YoD)

Once the required number of events is determined to reach a desired sensitivity level, the conversion from nEvents to PoT is simple and uses the cross sections. GENIE further simplifies this as it accounts for PoT in the simulation and will simulate the PoT required to produce the sample.

YoD can then be calculated from the PoT using the flux of the beam.

Determining years of data to achieve NuTeV sensitivity

Once we have the projection of sensitivity vs nEvents, a determination can be made as to how many events are required to achieve the sensitivity of the NuTeV experiment when we switch to LAr. Using the flux of the beam and the cross sections, we will be able to calculate the number of protons on target are required to achieve this sensitivity and finally the years of data required.

The table on the right is the data we plan to have by the end of the project. The goal would be to plot the relation between selection efficiency and the years of data necessary to match the NuTeV experiment sensitivity level.

Selection Efficiency (%)	Years of Data
40	
50	
60	
70	
75	
80	
83	
85	
87	
90	
95	
100	

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Conclusions

- Most measurements of $\sin^2\theta_W$ agree with the SM, but some experiments have produced results that contradict the other experiments and have a small enough uncertainty to be significant.
- Through neutrino experiments, we have been able to measure the weak mixing angle and compare it to the Standard Model to search for BSM physics
- DUNE is the flagship neutrino experiment and promises to shed light on many questions regarding neutrino physics
- While DUNE is already slated for construction in the early 2030s with wire plane detector technology, the QPix upgrade has already been proposed and is being tested and developed.
- We have simulated an experiment with QPix technology to project the sensitivity of measuring the weak mixing angle
- The hope is that this thesis will demonstrate enough motivation for this measurement to be done when the beam and detectors come online.

Backup Slides

References

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Muon depositing energy over pixels - Carter Eikenbary - [Reconstruction Methods with Q-Pix Simulations - Google Slides](#)

Pixel detector reconstruction cartoon - Ben Meleton

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Pixel Cartoon - combined from two figures in https://tobackgroup.physics.tamu.edu/wp-content/uploads/sites/21/2022/06/Asaadi_TAMU_Project.pdf

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Detector Response video clip - Ben Meleton -

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Pixel Reset Distribution - Carter Eikenbary -

https://docs.google.com/presentation/d/12rDmezmfA07RrJO4jtvpuhMEPVvGBrsUTSR9peFN6GU/edit#slide=id.g26ca487fac7_0_347

2D Event reconstruction - Carter Eikenbary -

<https://docs.google.com/presentation/d/1mXA18KEOvTj1iyE0IKKm7S8ljTnsBTEsB9EWBhj8dws/edit?usp=sharing>

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Reconstruction results - Carter Eikenbary -

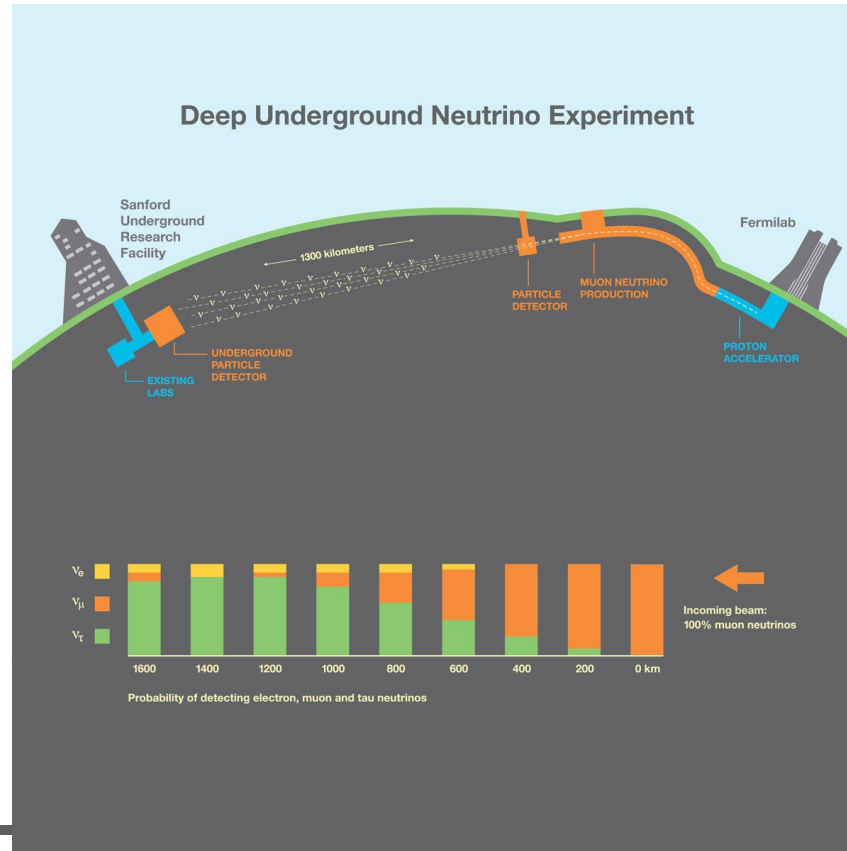
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Neutrino Fluxes at DUNE ND - <https://arxiv.org/pdf/2002.03005>

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Selection Efficiencies - <https://arxiv.org/pdf/2002.03005>

Neutrino Oscillation Requires experiment to be done with DUNE ND



https://particlephysics.ca/wp/wp-content/uploads/project_dune_02.jpg

Statistical Uncertainty expected to dominate

SOURCE OF UNCERTAINTY	$\delta \sin^2 \theta_W$	δR^ν	$\delta R^{\bar{\nu}}$
Data Statistics	0.00135	0.00069	0.00159
Monte Carlo Statistics	0.00010	0.00006	0.00010
TOTAL STATISTICS	0.00135	0.00069	0.00159
$\nu_e, \bar{\nu}_e$ Flux	0.00039	0.00025	0.00044
Energy Measurement	0.00018	0.00015	0.00024
Shower Length Model	0.00027	0.00021	0.00020
Counter Efficiency, Noise, Size	0.00023	0.00014	0.00006
Interaction Vertex	0.00030	0.00022	0.00017
TOTAL EXPERIMENTAL	0.00063	0.00044	0.00057
Charm Production, Strange Sea	0.00047	0.00089	0.00184
Charm Sea	0.00010	0.00005	0.00004
$\sigma^{\bar{\nu}}/\sigma^\nu$	0.00022	0.00007	0.00026
Radiative Corrections	0.00011	0.00005	0.00006
Non-Isoscalar Target	0.00005	0.00004	0.00004
Higher Twist	0.00014	0.00012	0.00013
R_L	0.00032	0.00045	0.00101
TOTAL MODEL	0.00064	0.00101	0.00212
TOTAL UNCERTAINTY	0.00162	0.00130	0.00272

[\[hep-ex/0110059v3\]](https://arxiv.org/abs/hep-ex/0110059v3) A Precise Determination of Electroweak Parameters in Neutrino-Nucleon Scattering (arxiv.org)